Idealization within a Structuralist Perspective*

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Abstract

This is a study of the concept of idealization in terms of the structuralist view of scientific theories. In the structuralist literature, the notion of idealization has been commonly analyzed as a sort of model construction. A short overview of this is provided in the first part of the paper. The main aim of the paper is however to provide a reconstruction of Leszek Nowak's account of idealization as a relation between structures. Other attempts at providing such an analysis are examined and compared. It will be shown that the present account amounts to considerable advantages. After a presentation of Nowak's syntactic approach and its main problems, I will try to reconstruct his analysis in structuralist terms. Although intuitively well motivated, Nowak's approach has some philosophical drawbacks associated to its essentialism, its syntacticism, and its counterfactual character. Our structuralist reconstruction will overcome these difficulties.

Keywords: idealization - Poznań approach - model construction - abstraction - counterfactuals - truth approximation

Resumen

El presente trabajo es un estudio del concepto de idealización en términos de la concepción estructuralista de las teorías. En la bibliografía estructuralista, la noción de idealización se ha analizado comúnmente en relación con la construcción de modelos. En la primera parte de este trabajo, se presenta un breve resumen de cómo ha sido tratada esta noción en el estructuralismo. El principal propósito del artículo, sin embargo, es proporcionar una reconstrucción, en términos estructuralistas, de la concepción de la idealización de Lezkez Nowak, uno de los filósofos más relevantes en el estudio de la idealización. En el artículo también se examinan y discuten otros intentos del estructuralismo de proporcionar tal reconstrucción, mostrándose cuáles son las principales aportaciones y ventajas de la presente. Tras presentar la concepción sintáctica de Nowak y sus principales problemas, se intenta llevar a cabo una reconstrucción de su análisis en términos de relaciones entre estructuras. Aunque intuitivamente bien motivada, la concepción de Nowak de la idealización presenta varios inconvenientes asociados a su esencialismo, su sintacticismo y su carácter contrafáctico. Nuestra reconstrucción estructuralista pretende superar todas estas dificultades.

Palabras clave: idealización - escuela de Poznań - construcción de modelos - abstracción - contrafácticos - aproximación a la verdad

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1. Introduction

Though there is an unanimous consensus that idealization is an usual resource in scientific reasoning and an essential aspect in the construction of scientific theories, there is by no means the same consensus in the use of the word "idealization" and there is no unique systematic (formal or informal) way to treat such idealizations and to understand their role in the structure of scientific theories. According to Leszek Nowak, undoubtedly one of the authors who has contributed a great deal to putting the term in the middle of the philosophical discussion, we can distinguish five different approaches to idealization:

(1) idealization is a method of transforming raw empirical data into "scientific facts";
(2) idealization is a method of constructing scientific notions;
(3) idealization is a deliberate falsification which never attempts to be more than truthlike;
(4) idealization is a way to create a construction that would fall exactly under the mathematical formalism serving as a model for the imprecise external world;
(5) idealization consists in abstracting or separating what is essential in the appearance of a phenomenon.[1]

These are however very informal and imprecise characterizations. For example, what is it meant by "scientific fact" in (1)? Or what is it meant by "truthlike" in (3)? Many papers and books in the philosophy of science literature from the sixties until now are devoted to this last question.[2] It is not my purpose here to discuss the notion of "truthlikeness" nor the vague one of "scientific fact", but to make precise the ubiquitous concept of idealization (more particularly, in structuralist terms).

Judging by the classification above, it could be said that the term "idealization" is an equivocal, ambiguous one and, in fact, there are many authors who advocate one or another idealization conception of the list from the point of view of different and sometimes seemingly opposed methodological approaches. But, although (1)-(5) have indeed led to different methodological proposals, I want to leave open the possibility of a mutual reconciliation of these five conceptions, because–according to their informal characterization–I do not see a contradiction among them.

Among the few formal (or semi-formal) attempts to reconstruct the notion of idealization, Nowak's is one of the most prominent.[3] Nowak's view has been followed, not only in Poland, by the members of the so-called Poznań-School, but also by well known philosophers as Nancy Cartwright (1989), who essentially agrees with him and adopts his account[4] and has influenced many others as

[1] This classification and the formulation of these five conceptions of idealization correspond almost literally to Nowak (1992), pp. 9-10.

[2] See Niiniluoto (1998) for an outline of the different accounts until the recent years.

[3] See Nowak (1980, 1989, 1990, 1991a, 1991b, 1992, 1995, and 2000).

[4] See essentially Cartwright (1989), pp. 202-204.

well.[5] Nowak sees in Galileo the introducer of idealization as the main method in the natural sciences (whereas Karl Marx would be the introducer of this method in the social sciences). In fact, many authors speak about "Galilean Idealization", understanding by that a process of counterfactual deformation or misrepresentation in the sense of consciously supposing that something false is the case for practical reasons (see McMullin 1985, Haase 1995 and, more recently, Weisberg 2007). We will offer a brief sketch of Nowak's account later in this article.

Another possibility of representing idealization in formal terms is given by the idea of understanding idealization in modal-theoretical terms (this is also accepted by Nowak 1991a), that is, by trying to represent counterfactual deformations as statements about possible worlds that are enough similar to a centered, fixed one, which is considered to be "the actual world". Following this line of thought, some philosophers use possible worlds and modal logic to provide an analysis of idealized generalizations. For example Cohen (1989), who sees idealization as a form of inductive reasoning, claims that idealized generalizations are to be graded inductively by means of the method of "relevant variables" in terms of their respective degree of "legisimilitude" and this notion of "legisimilitude" can be analyzed in its turn in modal-theoretical terms. Ronald Laymon is also among the most important philosophers who have concerned themselves with the problem of idealization. In his many contributions,[6] idealization is treated as a sort of approximation to the truth to a certain degree. Laymon (1982) explicitly admits a "converging counterfactual theory of confirmation", according to which scientists would proceed as if they were developing more and more accurate descriptions of reality: "convergence to experimental values given more realistic treatments is then seen as confirming the theoretical basis of these calculations" (Laymon 1982, p. 115). Laymon does not explicitly provide a formal account in terms of modalities, but this suggestion can surely be carried out, by providing an appropriate semantics for the counterfactual conditionals involved. A Lewisian semantics could do the job, but no one has provided a formal account of idealization explicitly in these terms. In his most formal contribution, Laymon (1987) uses lattices (Scott domains) to provide a model of approximation and idealization in the testing of scientific theories. Following Laymon's idea, Ibarra & Mormann (1994) present a reconstruction of the idealizations involved in the case of the law of simple pendulum in structuralist terms.

Following with the formal approaches to idealization that try to take the counterfactual aspects into account, Rott's contribution (1994) makes use of Peter Gärdenfors's concept of *epistemic importance* and its application to the problem of the rational changes of belief. With the aim of offering an adequate analysis of the non-monotone character of theory change, Rott suggests treating a scientific theory as a set of statements (just as the classical concept) plus a relation of "theo-

[5] The whole series of Rodopi's volumes entitled "Idealization", which belongs to the *Studies in the Philosophy of Science and the Humanities*, may serve as testimony.

[6] See Laymon (1980, 1982, 1985, 1987, and 1989).

retical importance", concept introduced to make possible minimal revisions within the content of a theory through counterfactual assumptions.

As interesting and fruitful as these approaches may be, I think they are not offering a general account of idealization in formal terms. Lattices (or Scott-domains), as used by Laymon (1987), may be helpful as a way of representing idealization and Rott's approach is surely interesting and fruitful in his attempt of giving a theoretical account of what we attempt to do in belief revision processes, particularly when counterfactual moves are involved with the aim of retaining part of the old and (partially) wrong theories. But they seem not good enough to capture the general aspects of idealization, mainly if we are interested in the nature of this concept. In my opinion, the counterfactual aspect of idealization is better to be captured in terms of a modal semantics, but a sufficiently accurate approach in these terms has not yet been provided. I must say that I am now working on this particular issue, but in the present paper I am more interested in providing a general discussion of the concept and a structuralist analysis of it, mainly as understood by Nowak, which has been one of the main advocates of the concept.[7]

My plan for the next is, then, the following. In the following section, I will present a first approach to idealization by distinguishing between different levels in which idealization can be realized in science. I will speak about a "hierarchy of idealization levels", but this hierarchy must not be understood in a restrictive way. Section 3 focuses on the role of idealization in model construction and, more particularly, on the issue of the data models, which has attracted the attention of several semanticist and structuralist authors. In section 4, I will present a brief sketch of Nowak's syntactic approach and try to reconstruct his analysis in structuralist terms. Finally, I will conclude with some general remarks.

2. Different levels of idealization. The structuralist view

The apparent divergence between the different concepts of idealization examined at the beginning of section 1 may be in part caused by the fact that some of these concepts capture *different levels of idealization*. Insofar as this is the case, there would not be contradiction between them. The present section is devoted to the presentation of what I call the "different levels of idealization". As was the case in the previous sections, we shall focus our attention on physics.

When one looks at the physics textbooks, it becomes clear that idealization takes place at different levels. They can be summarized in the following way, or at least I propose to do so:

(i) Selection of the relevant parameters: no system is really isolated from the rest of the world, but the great number of variables would make the investigation impossible, unless we neglect the influence of some of them. For

[7] For more reference to the counterfactual approach to idealization and its relation to Nowak's account see Mormann (2007), Niiniluoto (2007), and Shaffer (2007).

example, in the law of the pendulum, we do not take into account the position of the moon and in the law of the fall of the bodies we neglect air resistance.

(ii) Simplifications introduced in the relevant parameters, as when we are studying the movement of satellites and make the ideal assumption that the surface of the Earth is perfectly spherical and that the density is constant inside it.

(iii) Approximation among laws: we get new, more realistic versions of a law within a theory and the old law can then be considered as an "ideal case" of the new one, as in the case of the relation between the law for simple pendulum and its more concretized version that takes into account the amplitude. When the amplitude is small enough, it becomes a parameter that can be neglected.

(iv) Approximation between theories: old theories can be "derived" as ideal or limiting cases of the new theories. For instance, Kepler's Laws of Planetary Motion as a limiting case of Newton's Theory of Gravitation, or this last one as a limiting case of Einstein's relativistic version. In such "derivations", idealizations play an essential role.

(v) Simplification and approximation methods to deal with complicated equations for which it is very difficult to find analytical solutions: these include numerical methods and the approximation methods of governing equations.

Idealization in the sense of (i) and (ii) plays an essential role in what we may call "model construction" (construction of data models as well as of theoretical models) and is related to some important problems like the abstraction/idealization distinction, the non-isolability of systems and *ceteris paribus* clauses.[8] Idealizations in the sense of (iii) are ideal versions of more concretized laws within a theory and, in the sense of (iv), are usually laws of an old theory which can be seen as ideal cases of laws of a newer and more accurate theory. In the sense of (iii) and (iv), idealization meets approximation (and the structuralist notion of specialization). One can even ask whether such cases as that of Kepler-Newton can be reconstructed more as a case of idealization (as Rott 1994 or the authors of the Poznań School) or more as a case of approximation (as Balzer, Moulines & Sneed 1987). The answer is that such cases can legitimately be reconstructed as cases of approximation and idealization as well. Note that to consider a particular example as a case of (iii) or as case (iv) means to consider the ideal case as a particular law within a theory or better as a different theory. It depends on how we want to reconstruct the case. It is a matter of pragmatics whether we take the law of ideal gases as an ideal case of the more concretized version formulated by van der Waals within the same theory (thermodynamics) or as a case

[8] On the abstraction/idealization distinction see, for example, Harré (1970, 1989), Cartwright (1989), Dilworth (1989), Suppe (1989), pp. 94-96, Nowak (1990), Haase (1995), pp. 113-136, Hüttemann (1997), pp. 129-177, and Thomson-Jones (2005). On the relation of idealization to the isolation of systems see in particular Sklar (2000). And on the relation of idealization to *ceteris paribus* clauses see Kowalenko (2009).

of approximation between two theories: the old Boyle-Mariotte theory of ideal gases and the newer one of van der Waals' thermodynamics. According to the structuralist analysis of thermodynamics, Boyle-Mariotte's law is more accurately reconstructed as a theory-element of the theory-net of thermodynamics (the "ideal-gas simple equilibrium thermodynamics") and van der Waals' version, as another theory-element ("van der Waals' simple equilibrium thermodynamics") of the same theory-net. Actually the relation between these two theory-elements is reconstructed as one of specialization and not of approximation.[9] The Kepler-Newton case is reconstructed as a relation of approximation between two theory-elements, that of Kepler's theory of planetary motion and that of Newton's gravitational classical particle mechanics (more properly a theory-element belonging to another theory-net). Idealization has then to do with both relations.[10] Idealization in the sense of (v) includes all mathematical tools for simplifying equations and finding approximated solutions.

The above classification of five levels coincides to a great extent with that of Balzer, Moulines & Sneed (1987). They distinguish among (1) model construction approximation, (2) application approximation, (3) law approximation, and (4) intertheoretical approximation (Balzer, Moulines & Sneed 1987, p. 325). Besides the fact that they speak preferably about approximation, the only differences seem to be that we have added the (v)-level, included by the said authors in their type (3) and that we have not explicitly mentioned type (2) as a level of idealization; nevertheless this kind should be included in our levels (i) and (ii): just because we are aware of our idealizations by constructing data and theoretical models in the way we do it, we should recognize the idealized character of the application of our theory to reality. Type (2), application approximation, consists in subsuming a conceptually systematized collection of data, i.e., a data model of a particular phenomenon that we want to explain, under a theoretical law, that is, under an "actual model" in structuralist terms. Model construction approximation (type 1) has to do with the construction of a "potential model" or a "partial potential model" out of the data model. Type (1) may be termed "pre-theoretical approximation", because at this level only potential and partial potential models are considered. Types (2) and (3) are called "intratheoretical" to distinguish them from the type (4) or intertheoretical approximation. We have already said that the authors of *An Architectonic* put both approximation and idealization together, although when they speak about the relation between laws and theories, they use the term "approximation": only when they consider the model construction, they use explicitly that of "idealization". When they introduce the notions of theory-element, empirical claim or theory-net, they do it adding the adjective "idealized" to these labels in order to indicate that such notions are not realistic, because they do not take into account the fact that all empirical theories contain features of *approximation*.[11]

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[9] Balzer, Moulines & Sneed (1987), pp. 192-198.

[10] For a more recent account of the relation between idealization and approximation see Liu (1999).

[11] Balzer, Moulines & Sneed (1987), p. 89.

Moulines & Straub (1994), who start from the structuralist foundations as exposed in *An Architectonic*, explicitly distinguish between idealization and approximation, characterizing the first as any attempt to relate a scientific theory to the data, or to empirical reality, or even to another theory. The important idea is that idealization is more general than approximation: it is the attempt to relate the theory to the world through the construction of certain ideal structures. Approximation is more specifically viewed as a process of comparison of structures which are already idealized. Moulines (1996) begins with recognizing the plausibility of the assumption according to which idealization and approximation are more or less the same thing, the only difference being that they describe differently the same phenomenon: theoretical inaccuracy. In this sense, the idealization consisting in characterizing the Earth as a perfect sphere would be just a paraphrase to express that this characterization is a good approximation of the geodesic values we would obtain if we measured the distance from the centre of the Earth to its surface point by point.[12] But Moulines (1996) goes further and argues that this quasi-identification between the two concepts does not reflect all the possible (interesting) senses of idealization and right away proposes the same distinction found in Moulines & Straub (1994). His example now is the application of Newton's gravitation theory to the (real) planetary system. In order to apply Newton's theory we must make some idealizations, among them the assumption that the planets move as a set of particles on smooth paths. From the structuralist point of view, this means to reconstruct the planetary system as a potential model of Newton's theory. Another question would be whether the planetary system fulfils Newton's gravitation law, i.e., whether the planetary system is an actual model too. The last question is one of approximation and, as Moulines says, "[w]e may be successful in idealizing but fail in approximating".[13] The point of the structuralists is that approximation is a more specific notion, a kind of idealization, but there are other sorts. The point I am arguing here is therefore similar, because, according to the conception sketched above, approximation is one of the levels at which idealization may occur, just a kind of idealization. My main contribution will be to introduce a special relation between structures (not just a model construction) which is intended to capture Nowak's concept of idealization but turning into a structuralist notion what originally was thought as a syntactic relation between statements (laws). This work will be carried out in section 4.

3. Model construction and data systems

Recent contributions have revealed how complicated the relation between phenomena and theory may be.[14] For example, Bogen & Woodward (1988, 1992 and

[12] Moulines (1996), pp. 158-159.

[13] Moulines (1996), p.160.

[14] Bogen & Woodward (1988, 1992 and 2003), Mayo (1996), Cartwright (1999), Giere (1999), Woodward (2000), Bailer-Jones (2009).

2003) point out the difference between phenomena and data by highlighting that a large number of causal factors usually play a crucial role in experiments depending on the specific experimental setup, on the kind of apparatuses that are employed in the measuring procedures and on the different methods involved in data processing and data analysis. The idea that theories are not confronted directly with brute experience, i.e., with raw and unordered data, but with structures of highly idealized data, the so-called *data models*, constructed out of the analysis of the raw data, is traced back to Patrick Suppes (1962), who took an example from learning theory as a case study. Suppes restricts the data models to those aspects of the experiment which have a parametric analogue in the theory. Models of data are designed to incorporate all the information about the experiment which can be used in statistical tests of the adequacy of the theory. But, at the same time, the empirical testing of a theory involves other models and conditions at different levels: theories of error, models of experiment and experimental design, *ceteris paribus* conditions, which typically involve important idealizations (Suppes 1962, pp. 258-259). More recently, Mayo (1996) and Harris (1999) have resorted to a similar distinction and have equally argued for a hierarchy of models.

The idea of "data model", as introduced by Suppes in the context of the statistical analysis of data, was soon followed by the authors of the semantic conception of theories–as particularly defended by van Fraassen (1980) and Suppe (1989)–and by the advocates of the Structuralist View. More recently, van Fraassen (2008) has distinguished between data models and surface models and contends that theories are really confronted with surface models; this distinction is new (it does not appear in van Fraassen 1980). Data models are those that summarize the relative frequencies that we find in nature and surface models idealize this summary to replace those frequencies by measures with a continuous range of values (see van Fraassen 2008, p. 167). The structures that are embeddable in theoretical models are the surface models. Van Fraassen further distinguishes between observable phenomena and appearances. He introduces the triple distinction theory-phenomena-appearances as a result of defending the autonomy of experiment and a pragmatic account of scientific representation. The observable phenomena underlie the appearances and the appearances are the outcome of the measurement procedures as recorded in various data models. Appearances are said to represent phenomena and are embeddable in theoretical models.

In recent years, Moulines (2005 and 2007) has presented an illuminating study of the relation of idealization to model construction and the characterization of the data models. For Moulines, idealization should be distinguished from approximation: idealization has to do with model construction, whereas approximation is a relation between already constructed (idealized) models. Moulines's approach begins with an observation about what is the starting point of scientific research. Scientists seem to start with a particular experiential situation (*ES*) which at the very beginning is to be described in ordinary language. But it is because of the vagueness and uncertainty of identity criteria for *ES* that scientists should refine and modify these descriptions of *ES* by constituting an operational base (*OB*) for *ES*. By means of such an *OB*, which mainly consists of systematic and public observation and manipulation of medium-sized objects, *ES* is transformed into an intersubjectively controlled experiential situation (*ICES*). It is important to see that the connection between *ES*, *OB* and *ICES* is not a necessary one and that there is no one-to-one correspondence even among *OB* and *ICES*: the same *OB* (or a different one) may serve to determine other *ICES*s. The essential is that the scientific community has decided by an intersubjectively control led process to transform the *ES* into a certain *ICES*. The construction of *ICES*s then depends on the community of scientists, or on a group of them. *ICES*s are to be codified and represented by correspondent data models. This also takes place within the community of scientists: it is the community that decides (partly by convention, partly by normalized and established procedures) to accept certain axioms in virtue of which the *ICES*s are represented in a certain way. So the construction of the data models has to do with the presentation of certain axioms or statements which are to be admitted by convention. These axioms include specifications about how the universe of discourse of our data models must be. The group of scientists surely has elaborated a theory to be identified with the class of its models, **M(T)**. For any structure $x \in$ **M(T)**, x should satisfy the laws of the theory. The application of theory **T** to reality cannot be done in a direct way, but via data models. Once we have constructed the data models, our task consists of trying to embed them as substructures of the theory.

The elements and different steps of the process can then be summarized in the following way (see Moulines 2005, pp. 326 and ff. and Moulines 2007, pp. 261 and ff.):

(1) scientists are confronted with an experiential situation (*ES*)
(2) scientists redefine the *ES* through an operational base (*OB*)
(3) scientists become an intersubjectively controlled experiential situation (*ICES*)
(4) construction of the data model corresponding to the *ICES*
(5) embedding of the data model under the theory, which is synonymous with the claim “the *ICES* can be subsumed under the theory”.

Let us illustrate Moulines’ account with examples. By conceptualizing the *ES* as a certain *ICES* and by constructing a data model corresponding to this *ICES*, the scientist goes from the ordinary language to the theoretical one and tries to construct from the hard data a corresponding data structure which can be embedded into his theory. By the process of conceptualization, idealizations and approximations usually take place. In the case of classical particle mechanics, macroscopic bodies are seen as particles without taking into account their color, form or magnitude (extension), their paths are considered continuous and their position are to be represented as points in a vector space. In the original description theoretical concepts do not take place, though the concepts we use may be theoretical with respect to other theories presupposed by the theory we now use and try to test. In structuralist terms, we first construct the data models and then we try to subsume them under the partial potential models of our theory. Finally, we extend the structures obtained in this way, if it is possible, as actual models of the theory. The relation between the data model and the partial potential models of our theory should be that of the model-theoretical concept of substructure. Recall that a structure $\mathbf{M'} = \langle D'_1, ..., R'_n \rangle$ is a substructure of $\mathbf{M} = \langle D_1, \ldots, R_n \rangle$ iff $D'_1 \subseteq D_1, \ldots, R'_n \subseteq R_n$, where some components of the substructure $\mathbf{M'}$ may be empty. Data models are always finite and what we do is to state the hypothesis that our theory can subsume these data. The idea behind our hypothesis is that for each data model $\mathbf{m_D} \in \mathbf{M}_D$ from the class of the data models there is an actual model $\mathbf{m} \in \mathbf{M(T)}$ such that **m** can be made to correspond to $\mathbf{m_D}$ in the sense just specified. This relation between structures can be blurred and, then, what we have is an approximation version of the same statement relative to certain metrics d and degree of accuracy ε, namely: for each data model $\mathbf{m_D} \in \mathbf{M}_D$ from the class of the data models there is an actual model $\mathbf{m} \in \mathbf{M(T)}$ and there is an approximation $\mathbf{m^*}$ to m such that $d(\mathbf{m^*}, \mathbf{m}_D) \leq \varepsilon$ and $\mathbf{m^*}$ can be made to correspond to $\mathbf{m_D}$ in the appropriate sense. This idea can be found more precisely formulated in Balzer (1997), chapter 3.[15]

Some authors have questioned the idea that models of the phenomena are arrived at as deidealizations of theoretical models.[16] It is well-known that the same data model can be embedded into different theoretical models, but this fact cannot be confused with the idealization-concretization process at the theoretical level. To invoke the first fact, which leads to the problem of underdetermination of the theory by the data, cannot serve as an argument against the idealization-concretization process and against the idea that as more correction terms are introduced, the (theoretical) model becomes more realistic (in relation to the data model)–just as Cartwright, Shomar & Suárez (1995) seem to argue.[17] The problem of underdetermination has been used as a weapon against scientific realism and in fact can be used to argue against the thesis that the idealization-concretization method provides us with an argument in favor of scientific realism, but not against the method itself as a way of approaching the data structures in a more realistic way. On the other hand, there are interesting answers to the anti-realistic arguments based on the underdetermination problem which give some hope to the realists.[18]

4. Nowak's account of idealization and its structuralist reconstruction

Nowak's account of idealization, which is the core of the Poznań-School approach, was presented as a book in English for the first time in 1980 as a systematic

[15] See also Balzer, Moulines & Sneed (1987), chapter 2 for the concept of partial potential model and chapter 7 for the relation of approximation.

[16] See for instance Cartwright, Shomar & Suárez (1995).

[17] Cartwright, Shomar & Suárez (1995), p. 142. See also Morrison & Morgan (1999).

[18] See Psillos (1999), Chapter 8. A well-known paper about the problem is Laudan & Leplin (1991).

interpretation of the Marxian idea of science, but it goes back to his many English and Polish papers published in the seventies. Although not belonging to the School, Krajewski's (1977) contribution to the study of the correspondence principle is very close to the ground ideas developed by Nowak and his followers. Further contributions were Zielińska's (1990), Kupracz's (1992) and Paprzycka's (1992). Nowak's approach was also very criticized in Poland and gave raise to a vivid controversy.[19] Nowak received two sorts of criticism: on one side, some Marxist authors criticized him–perhaps with a certain dogmatism–for misinterpreting Marx's own ideas on methodology; on the other, he was criticized on philosophical grounds and for logical inaccuracies.[20] Only this second sort of criticism will deserve our attention later.

An excellent survey of the idealizational approach to science is Nowak's (1992), which presents an overview of the different contributions and applications to case studies. Nowakowa's (1994) is also a suitable introduction to the main ideas of the School. Nowak & Nowakowa (2000) collects many papers of Nowak, his wife or even of both, some of which had not appeared in English earlier, or at all.[21] The book contains also an exhaustive bibliography on idealization. Brzeziński *et al.* (2007) presents a recent collection of different essays presented in honor of Prof. Nowak. All of them discuss different aspects of his philosophy, but the book is mainly dedicated to the concept of idealization.

Nowak and his followers adopt a syntactic view on idealization by analyzing it as a relation between lawlike statements. A law (in the present context) will typically have the form of a universally quantified statement (Nowak and followers use first-order logic, but this is not an essential point) and theories are supposed to be classes of such statements. Nowak's account of the idealization-concretization process can be presented in the following way. We can start from the most idealized law (T_k) to the least one (T_0). Let be x to denote a given "real system" (in Nowak's words). "Real systems" are here to be understood as individuals belonging to the domain of the theory: real systems are (particular examples of) conductors in the case of the theory of electricity or gases in the case of the theory of gases. They then constitute the ontology of the theory. Let $F(x)$ be a magnitude or a quality of x, $H_1(x),\ldots,H_n(x)$ some parameter functions which $F(x)$ *necessary* depends on, $p_k,\ldots p_1$ the parameter functions which can be taken into account in the progressive concretizations of the most idealized law T_k, $f_k,\ldots, f_0$ the laws through which we can determine the value of $F(x)$, and assume that x belongs to a class R which constitutes the empirical domain of that system and that all these functions are real-valued. When Nowak says that $H(x)$ plays the role of a *necessary* factor, he is assuming that there are factors which cannot be neglected, which *necessarily* (in some sense of necessity) determine the value of $F(x)$. He does not specify

[19] Unfortunately almost all the authors who took part in the discussion wrote their contributions only in Polish.

[20] See Krajewski (1977), p. 22.

[21] Nowak (1992) is also included in the book (see pp. 109-184).

what he has in mind and in fact this is one of the critical points which have been noticed by his commentators. Essentialism is a characteristic aspect of Nowak's position. He ascribes this distinction between essential and adventitious factors to Marx's *Capital* and considers it a fundamental feature of the method of idealization.[22] This is a first controversial point, because it seems clear to me that we can support an idealizational approach without defending any such metaphysical thesis. According to Nowak, there are usually more than one essential factor (maybe many) and even among the essential factors a hierarchy can be distinguished according to their degree of influence.[23] For the sake of simplicity, let us suppose that there is only one of such necessary factors.

Returning to our point of departure: the structure of idealized laws is that of a conditional statement, in which the antecedent contains some idealizations (which typically have the form: for certain x certain parameter function assumes the value 0, 1 or infinity). We have here a second point of controversy, namely that of the interpretation of such conditionals. If we assume they are material conditionals, then we come to the well-known problem that they are always (trivially) true (because ideal conditions are false). So it seems more natural to interpret such conditionals as counterfactuals.[24] But this does not seem to be Nowak's interpretation and on the other hand neither he nor his followers present any analysis of such counterfactuals at all. The most idealized law, T_k, contains k idealizing or counterfactual assumptions: $p_k(x) = 0, \ldots, p_1(x) = 0$. The next, less idealized one, T_{k-1}, must contain $k-1$ idealizing assumptions, etc. T_i is said to be "the immediate idealization" of T_{i-1} and, conversely, T_{i-1} is "the immediate concretization" of T_i:[25]

$$(T)\ F(x) = f_k(H(x)).$$

$$(T_k)\ R(x) \wedge p_1(x) = 0 \wedge \ldots \wedge p_{k-1}(x) = 0 \wedge p_k(x) = 0 \Rightarrow F(x) = f_k(H(x)).$$

$$(T_{k-1})\ R(x) \wedge p_1(x) = 0 \wedge \ldots \wedge p_{k-1}(x) = 0 \wedge p_k(x) \neq 0 \Rightarrow F(x) = f_{k-1}(H(x), p_k(x)).$$

$$(T_{k-2})\ R(x) \wedge p_1(x) = 0 \wedge \ldots \wedge p_{k-2}(x) = 0 \wedge p_{k-1}(x) \neq 0 \wedge p_k(x) \neq 0 \Rightarrow F(x) = f_{k-2}(H(x), p_k(x), p_{k-1}(x)).$$

....

$$(T_i)\ R(x) \wedge p_1(x) = 0 \wedge \ldots \wedge p_i(x) = 0 \wedge p_{i+1}(x) \neq 0 \wedge \ldots \wedge p_{k-1}(x) \neq 0 \wedge p(x) \neq 0 \Rightarrow F(x) = f_i(H(x), p_k(x), \ldots, p_{i+1}(x)).$$

....

$$(T_0)\ R(x) \wedge p_1(x) \neq 0 \wedge \ldots \wedge p_k(x) \neq 0 \Rightarrow F(x) = f_0(H(x), p_k(x), p_{k-1}(x), \ldots, p_1(x)).$$

Obviously this scheme does not cover all forms of idealizing laws, but capture the general idea of the process. There are two important points to be noticed. One is the interpretation of '$\Rightarrow$'. If we interpret this connective as a material conditional, as in fact it seems that Nowak and his followers have done, idealizations become vacuously true statements. It seems therefore that the conditional must

[22] Nowak (1980), p. 95.

[23] See Nowak (1980), pp. 97-98.

[24] Compare Niiniluoto (1999), p. 137.

[25] These are not Nowak's terms, but mine, because I interpret that the idealization relation is transitive.

be counterfactual.[26] The second point is the fact that it is surely intended that the idealization relation is transitive, that is, T_k is an idealization of T_{k-1} but also of T_{k-2}, which is the immediate concretization of T_{k-1}. It also seems that Nowak's characterization implies a linear sequentiality of factors which are supposed to be ordered according to their degree of importance. But, if my interpretation is correct, Nowak's analysis would become too restrictive, because we can neglect whichever of the factors we think to be negligible in a given time. Nowak's characterization can also be interpreted in the sense that it is a reconstruction made *a posteriori* by the philosopher of science, who simply orders the factors just according to the chronological order in which they were discovered by the scientist. According to the Poznań-School Conception, T_{k-1} pass asymptotically into T_k when for each x, $p_k(x)$ tends to zero.[27] Normally, even T_0 is not met in empirical sciences. Instead, the scientist introduces corrections in order to approximate the value of F. In this case, we introduce approximations of idealizations instead of concretizations.

Nowak's account of idealization is certainly an interesting achievement in philosophy of science and his idea is intuitively valid. This notwithstanding, Nowak's approach has some difficulties. We are going to see now the problems which–in my opinion–render unsatisfactory the Poznań-School conception of idealization. The objections can be summarized in the following way:

(1) The Poznań-School approach to idealization is too much depending on the classical view of scientific theories. It is true that, for instance, Nowak (1972) criticizes Nagel's account as inadequate and incomplete on the ground that his model does not cover idealizations as an important part of scientific method, but only on an empirical basis and that, for Nowak, this is so because of the positivistic view to which Nagel is still subjected. But it is also true that Nowak's idealization presupposes the classical conception of theory according to which theories are classes of statements and that idealization is basically reconstructed as a relation between statements. Our first objection is surely not a problem *per se*, but it becomes a difficulty for all those philosophers who argue that to treat scientific theories just as classes of statements is an insufficient or even a wrong way of reconstructing their logical structure.

(2) Nowak's analysis seems to be not sufficiently general, since the neglected parameters can take other values than 0, and not sufficiently accurate, because it does not say, for instance, how the functions f_k and f_{k-1} are related for one to be part in an idealization of the other.

(3) If my interpretation of Nowak is correct and it presupposes a linear ordering of idealizations-concretizations, that is, it presupposes that there is a linear ordering of parameters according to their influence, then his characterization does not seem to be accurate. Why should it not be legitimate to take a different parameter for the idealization (for example, if we do not

[26] See, for instance, Niiniluoto (1999), p. 137.

[27] Nowak does not specify any metric or topology.

know how much influent it is). It also presupposes a notion of essentiality of factors which, as Paprzycki & Paprzycka (1992) have correctly pointed out, needs at least a more precise definition. Nowak's further ideas presuppose also a distinction between ideal and real objects and leads to a metaphysical conception of science which is, in my opinion, something suspicious.[28]

(4) Since ideal conditions are manifestly false, we can not use the material conditional to formalize an idealizational law, but a counterfactual conditional. But this leads to the well-known problem of the analysis of such conditionals. Nowak and his followers do not seem to say anything about this.

(5) Nowak and his followers do not seem to recognize the role of approximation as another important relation between laws, but rather as a method of comparison of the values of the parameters appearing in the laws.

(6) My last objection is the most important one and has to do with the fact that Nowak's analysis does not seem to grasp well enough the difference between an intratheoretical idealization and an intertheoretical one. His analysis seems to be more an attempt at characterizing the relation of intratheoretical idealization (as idealization between lawlike statements of a given theory) or at least it is not enough clear which theory each idealizational statement belongs to. In which theories these statements should be satisfied?

For other kinds of criticisms see Krajewski (1977) and Haase (1995).[29] After having sketched my objections to Nowak's idealizational approach, I want to focus the rest of the section on the reconstruction of Nowak's ideas in structuralist terms.

Nowak (1989), (1990) and (1991) presents an analysis of the phenomenon of idealization in terms of the notion of counterfactual deformation. Ibarra & Mormann (1994) reconstruct Nowak's idea of counterfactual deformation by introducing it as a relation between structures: (potential) models are presented not in the usual way, but adding to the base sets and relations a new component which consists of the carriers $U_1, \ldots, U_p$ of the relations $f_1, \ldots, f_p$ so that each f_i is a subset of U_i. Potential models are then of the form: $x = \langle A_1, \ldots, A_n, f_1, \ldots, f_p, U_1, \ldots, U_p \rangle$. We can then define:

Def. 1: If $x = \langle A, f, U \rangle$ is a (potential) model, then x' is a *soft counterfactual deformation* of x iff there exists f' such that $x' = \langle A, f', U' \rangle$, and x' is a *hard counterfactual deformation* of x iff there exist f' and U' sucht that $x' = \langle A, f', U' \rangle$, where $U \neq U'$.

The relation of concretization/idealization is defined as a relation between theory-elements in terms of that of counterfactual deformation operator in this way:

[28] See Krajewski (1977), pp. 25-28.

[29] See Krajewski (1977), chapter 1; Haase (1995), chapter 2, section 2.1.4, in particular p. 101 for a survey.

Def. 2: If $\mathbf{T} = \langle \mathbf{K}, \mathbf{I} \rangle$ is a theory-element, $\mathbf{K} = \langle \mathbf{M}, \mathbf{M_p}, \mathbf{M_{pp}}, \mathbf{r}, \mathbf{C} \rangle$, where $\mathbf{M_p}$ has as elements structures of the form $\langle A, f, U \rangle$, a *counterfactual deformation operator* $\mathbf{d}$ of $\mathbf{T}$ is a map $\mathbf{d}: \mathbf{M_p} \rightarrow \mathbf{M_p}$ of the following form:

(1) $\mathbf{d}(\langle A, f, U \rangle) = \langle A, f', U' \rangle$, where f' and U' are as above
(2) $\mathbf{dd} = \mathbf{d}$
(3) $\mathbf{d}[\mathbf{M}] \subseteq \mathbf{M}$

U can be identical to U' or not, so that $\mathbf{d}(\langle A, f', U' \rangle)$ can be either a soft counterfactual deformation or a hard one. Conditions (i) and (ii) seem well motivated. Condition (ii) says that $\mathbf{d}$ is idempotent: the double application of a counterfactual deformation operator does not yield anything new. But what about condition (iii)? The idea is that $\mathbf{d}$ should be a projection which preserves the actual models. They argue: "The purpose of counterfactual deformation is to transform a 'good' potential model into an actual one" (Ibarra & Mormann 1994, p. 187). The role of counterfactual deformation operators is then "to eliminate certain factualities that hinder potential models from being actual ones" (Ibarra & Mormann 1994, p. 187). They are thinking of examples such as that of assuming that the bob of a pendulum is a mass-point. This seems to be the case within this example, but there are other examples of idealization in which our ideal conditions contradict the axioms of the theory. For instance, the supposition made in relativity physics (by Einstein and others) according to which there are rigid bodies (in the classical sense). This contradicts the principles of relativity. Newton neglected the influence of the attraction of the Sun by the planets despite the fact that it contradicts his actio-reactio principle. So I think condition (iii) is only acceptable for a particular kind of counterfactual deformations: those which are used to characterize the theoretical domain and are contingently false with respect to the actual facts, just as in the case of the pendulum. It is interesting to make an observation here. The case of the supposition of the mass-point in classical mechanics is self-contradictory only when we have the concepts of gravitational potential and gravitational mass. Newton was aware of the impossibility *of fact* of the mass-points, but his argument is only philosophical:

> The qualities of bodies, *which admit neither intensification nor remission of degrees*, and which are found to belong to all bodies *within reach of our experiments*, are to be esteemed the universal qualities of all bodies whatsoever. [...] the extension, hardness, impenetrability, mobility, and inertia of the whole, result from the extension, hardness, impenetrability, mobility, and inertia of the parts (Newton 1687[1934], vol. II, pp. 398-399; my italics).

Ideal conditions that strictly speaking condradict the theory in which such suppositions are made are typical when we try to derive an old theory from the new, more precise one. Newton made them when he attempted to derive Kepler's laws from his own theory. Ibarra and Mormann's reconstruction then seems not suitable for intertheoretical cases in particular.

Ibarra & Mormann (1994) suggest to assume that the set of counterfactual deformation operators for a theory-element is (at least) a semilattice, that is, that the composition of $\mathbf{d}$ satisfies the property of commutativity. Ibarra and Mormann also require idempotence and associativity (which is always guaranteed in the composition of functions). This suggestion seems also well motivated and tries also to mirror Laymon's account. They then define:

Def. 3: A *theory-element* $\mathbf{T}$ *with an idealization structure* $\mathbf{D}$, denoted by $\langle \mathbf{K}, \mathbf{I}, \mathbf{D} \rangle$, is a theory-element $\langle \mathbf{K}, \mathbf{I} \rangle$ endowed with a semilattice $\mathbf{D}$ of counterfactual deformation operators defined on $\mathbf{M_p}$. Provided a partial order $\langle \mathbf{D}, \leq \rangle$ between the elements of $\mathbf{D}$, where $\leq$ is a relation defined in this way: $\mathbf{d} \leq \mathbf{d}'$ iff there is a $\mathbf{d}'$ such that $\mathbf{d} = \mathbf{d}'$, $\langle \mathbf{D}, \leq \rangle$ induces in a natural way a partial order on $\mathbf{M_p}$: $x \leq y$ iff there is a $\mathbf{d} \in \mathbf{D}$ with $\mathbf{d}(x) = y$. The idea behind it is that we want to become more idealized theories by applying progressively more counterfactual deformation operators: if $x \leq y$, we then say that y is an idealization of x or, conversely, that x is a concretization of y.

Def. 4: If $\mathbf{T} = \langle \mathbf{K}, \mathbf{I}, \mathbf{D} \rangle$ and $\mathbf{r}' = \langle \mathbf{K}', \mathbf{I}', \mathbf{D}' \rangle$ are theory-elements with idealization structures, $\mathbf{T}$ is called a *concretization* of $\mathbf{T}'$ (we write: $\mathbf{T} \leq \mathbf{T}'$) iff:

(1) $\mathbf{K} = \langle \mathbf{M}, \mathbf{M}'_{\mathbf{p}}, \mathbf{M}'_{\mathbf{pp}}, \mathbf{r}' \rangle$, $\mathbf{K}' = \langle \mathbf{r}', \mathbf{M}'_{\mathbf{p}}, \mathbf{M}'_{\mathbf{pp}}, \mathbf{r}' \rangle$
(2) $\mathbf{D} \subseteq \mathbf{D}'$
(3) for all potential models $x \in \mathbf{M}'_{\mathbf{p}}$ for which there is a counterfactual deformation operator $\mathbf{d}' \in \mathbf{D}'$ such that $\mathbf{d}'(x) \in \mathbf{M}'$ there is a counterfactual deformation operator $\mathbf{d} \in \mathbf{D}$ with $\mathbf{d} \leq \mathbf{d}'$ such that $\mathbf{d}(x) \in \mathbf{M}$.
(4) $\mathbf{I} = \mathbf{I}'$.

(I have introduced some small changes in Ibarra and Mormann's definitions.)

Let us briefly comment both definitions. Conditions (2) and (3) of *Def.* 4 should preclude that the counterfactual deformations of the idealized theory are stronger than those of the concretized theory and that is well motivated. But conditions (1) and (4) are too strong, if we want–as I think Nowak does–to take intertheoretical cases into account, because the new, more precise theories have extended their set of intended applications: think of the Newton-Kepler case, where Newton's theory is intended to be applied not only to planetary systems–as in the case of Kepler's celestial mechanics–but to all mechanical systems. (1) is also too strong, because the new theories usually have also extended their components, they are even structures of a different similarity type, as in the Kepler-Newton case. I believe Nowak and his followers attempted at covering also these cases. Therefore, I conclude Ibarra and Mormann's analysis is not even appropriate as a reconstruction of Nowak's ideas. Recently, Mormann (2007) explores the relation between idealization, representation and counterfactual deformation in the framework of a possible worlds-account. I will not discuss his approach in the present article, because it has not explicitly to do with the aim at providing a structuralist reconstruction of Nowak's ideas.

Similarly, Haase (1995) and (1996) has provided a structuralist analysis of the notion of idealization which is very close, and in fact is also based upon, Nowak's typology of deformational procedures. Both idealization and abstraction are for Hasse the most important methods of what she generally calls (in german) "Veränderungen von Repräsentationen" (alterations of representations). For Haase, representations can be of a formal nature: formalizations of a theory in a given language which is sufficient for representing all the complexity of the theory, but there can also be mental, "experimental" (which could be formulated as physical models of a certain kind) and even other kinds of representation–Haase is not very precise in this point–. In the present context, the important point is that to each representation always corresponds a universe of discourse. She then defines a relation of idealization to hold between universes in the following terms:[30] $R^{ij} \subseteq \mathbf{U}^{\mathbf{i}} \times \mathbf{U}'^{j}$ (for $1 \leq i \leq m$ and $1 \leq j \leq n$) is an *idealization relation* iff there are families of universes $\mathbf{U}$ and $\mathbf{U}'$ such that $\mathbf{U}$ is a family of non-empty and non-idealized universes–that is, universes to which it has been applied an operation of idealization in the sense of Nowak–, $\mathbf{U}'$ is a family of non-empty idealized universes, and $\mathbf{U} \cap \mathbf{U}' \neq \varnothing$. This means that for the objects within the universes of the families related by idealization must hold the counterfactual deformation method of idealization in the sense of Nowak. Haase distinguishes between a non-idealized representation (RI) and an idealized representation (RII) and defends a pragmatic conception of Galilean idealization. In this sense, her definition of "idealized representation" is more subtle, because it includes the reference to the scientific community of a certain period in the history of science and the concept of intention. In such terms, she defines two relations between representations: one of idealization and the other of abstraction, the main difference being that the relation of abstraction is made in terms of a selection of characteristics ("Merkmale") or properties (parameters if you want) just as in case of Nowak. Whereas the relation of idealization implies the application of a counterfactual deformation procedure, that of abstraction does not requires that.[31] The same objections could be raised to Haase's analysis as in the case of Nowak's new account. The principal objection would be that it does not seem accurate enough to reduce the idealization relation between two theories merely to a relation among their universes of discourse, because the deformation procedures are to be applied to the specifications of the theoretical domains and these specifications usually belong to the axioms of a theory. Idealization as intertheoretical relation involves therefore more a relation between statements (as in the case of the Poznań School) or between the statements used to determine a certain class of structures (if we follow the structuralist approach). But, in any case, a positive contribution of Hasse's account seems to be the incorporation of a pragmatic element which surely should to be taken into account in a more complete analysis of idealization. These pragmatic aspects of idealization can easily be incorporated in a structuralist version of Nowak's idealization.

Comparisons between the Poznań-School approach to idealization and the structuralist conception have been made by Kuokkanen (1988), Hamminga

30 Haase (1995), p. 124.

31 Haase (1995), pp. 128-129.

(1989), Kuokkanen & Tuomivaara (1992) and Balzer & Zoubek (1994). See also Haase (1995) and (1996). As I cannot here comment all of them, I will only present my own structuralist reconstruction and results, which are in part based on the work of these authors. I am particularly indebted to Balzer & Zoubek (1994) and Kuokkanen & Tuomivaara (1992). I will later mention the main differences between the account of these authors and my own account. In what follows I suppose a certain familiarity with the structuralist methods.

Let θ be the similarity type for structures of the form: $\langle D_1',..., R_n' \rangle$ is a substructure of $\mathbf{M} = \langle D_1,\ldots, D_r, A_1,\ldots, A_m, R_1,\ldots, R_n, p_1,\ldots, p_i\rangle$, where $D_1,\ldots, D_r$ are base-sets, $A_1,\ldots, A_m$ are auxiliary base-sets, $R_1,..., R_n$ are functions, relations or constants over $D_1,\ldots, A_m$, and $p_1(\overline{x}),\ldots, p_i(\overline{x}) \in \mathbf{R}$ (where $0 < i < k$ and $\mathbf{R} = A_j$, for some $j = 1,..., m$), where $p_1,..., p_i$ are distinguished parameter functions which can be neglected under certain conditions, or what is the same, the different factors introduced by the successive concretizations. The domains of these functions are subsets of some of the base-sets $D_1,\ldots, D_r$. Let θ' be an expansion of θ, $\langle D_1,\ldots, D_r, A_1,\ldots, A_m, R_1,\ldots, R_n, p_1,\ldots, p_i, p_{i+1}\rangle$, in order to include one factor more. Let $\mathbf{T}$ be a given theory understood as a certain class $\{m: m = \langle D_1,\ldots, D_r, A_1,\ldots, A_m, R_1,\ldots, R_n, p_1,\ldots, p_i\rangle\}$ which satisfies some axioms linking the different components of the θ-structures. Let now $\mathbf{T}'$ be another theory, understood in the same way, as a certain class of θ-structures. Let $\mathbf{M(T)}$ and $\mathbf{M(T')}$ be the classes of those structures which satisfy the axioms of $\mathbf{T}$ and $\mathbf{T}'$ respectively, and $\mathbf{I(T)}$ and $\mathbf{I(T')}$, the classes of the intended applications of $\mathbf{T}$ and $\mathbf{T}'$, respectively. In $\mathbf{M(T')}$, p_{i+1} takes a value greater than 0 (for each x). Let *red*: $Str(\theta')\rightarrow Str(\theta)$ be the "reduct" function which for each $m = \langle D_1,\ldots, D_r, A_1,\ldots, A_m, R_1,\ldots, R_n, p_1,\ldots, p_i, p_{i+1}\rangle$ gives the structure $red(m) = \langle D_1,\ldots, D_r, A_1,\ldots, A_m, R_1,\ldots, R_n, p_1,\ldots, p_i\rangle$.

Let $\mathbf{M}^* := \{\langle D_1,\ldots, D_r, A_1,\ldots, A_m, R_1,\ldots, R_n, p_1,\ldots, p_i, p_{i+1}\rangle \in \mathbf{M_p(T')}: \forall \overline{x} (p_1(\overline{x}) \geq 0 \wedge ... \wedge p_{i+1}(\overline{x}) \geq 0)\}$.

I disagree with Balzer's and Zoubek's reconstruction in a first point: while they take distinguished values (real numbers) of the parameter functions as components of the structures, I prefer to take the parameter functions themselves. So $p_i,\ldots, p_{i+1}$ are the parameter functions which we can idealize and $R_1,..., R_n$ the main relations and functions which are necessarily to be taken into account (in Nowak's terminology). The difference between main or essential factors and secondary factors (those that can be successively neglected, i.e., that can be "idealized") is also presupposed in Kuokkanen & Tuomivaara (1992, pp. 79-80). But this sort of essentialism could be easily skipped from the present account.

Then we can define:

Def. 5: $\mathbf{T}$ is an *idealization* of $\mathbf{T}'$ (and hence that $\mathbf{T}'$ is a *concretization* of $\mathbf{T}$) iff:

(i) $\mathbf{I(T)} \subset red[\mathbf{I(T')}]$

(ii) $red[\mathbf{M(T')}] \cap \mathbf{M(T)} = \varnothing$

(iii) $\forall m \in Str(\theta)(m \in \mathbf{M(T)} \leftrightarrow \exists m^*(m^* = \langle D_1,\ldots, D_r, A_1,\ldots, A_m, R_1,\ldots, R_n, p_1,\ldots, p_i, p_{i+1}\rangle \in \mathbf{M(T')} \wedge \forall \overline{x} (p^{m^*}_{i+1})(\overline{x}) = 0) \wedge red(m^*) = m)$.

The essential point in this structuralist reconstruction of the Poznań account is that, instead of classes of formulas, we have classes of models. The models of the concretized theory contain the parameter function which is taken for the first time into account, while in the models of the corresponding idealized theory this same parameter function turns out to be a constant function which always yields the value zero (or another constant value) and hence it becomes to be negligible. Then the process of concretization can be reconstructed in the following way: We can get the non-idealized theory $\mathbf{T}'$ as the class of structures which take into account the new parameter from the idealized theory **T**, which is viewed as the reduction by the function *red* of the class of structures $\mathbf{T}'$. This transition must satisfy conditions (i)-(iii). Condition (i) says that the intended applications of the idealized theory are exactly the same of the concretized theory, except only that this second theory takes a new factor into account; this means that the concretized theory has more intended applications than the idealized one. Condition (ii) says that the concretized theory and the idealized one are dramatically different with respect to their actual models, although they coincide entirely with respect to their potential models (the potential models of the idealized theory are simply reductions by *r* of the potential models of the concretized theory). This is so, because according to the concretized theory it is not possible for a real system to be a model of the concretized theory without taking into account a certain relevant factor, which is not taken into account by the idealized theory. Condition (iii) says that $\mathbf{M}(\mathbf{T})$ can be seen as the "reduct" of that part of $\mathbf{M}^*$ in which the new parameter equals zero, being the laws of $\mathbf{M}(\mathbf{T})$ the same as those of $\mathbf{M}(\mathbf{T}')$ in this special case. A similar counterpart is lacking in Kuokkanen & Tuomivaara's (1992) approach. The main difference between their and my own account is that, in my case, idealized structures can be obtained as reducts of more concretized structures by means of an idealized assumption consisting in stating that certain factor can be neglected, whereas in the case of Kuokkanen and Tuomivaara the same factors are always present in the structures (those concretized and those idealized as well). It is an alleged advantage of my account that it can represent the idealization process by means of an operation (the "reduct" function) and, at the same time, to make clearer the difference between idealization and de-idealization at the level of the mathematical structures. As a reconstruction of Nowak's approach my account reveals to be better too, because idealized and concretized structures can be easily presented in this way as those structures satisfying certain idealized and concretized statements as formulated by Nowak. Now let us see how this reconstruction allows us to overcome the main difficulties associated with Nowak's approach. Nowak's essentialism can easily be skipped from the structuralist account, as the idealization relation is a relation between structures already available, i.e., previously constructed. The idealization can then be seen basically as a relation between theory-elements just as any other intertheoretical relation. And no further essentialist consideration should necessarily be introduced in this framework. The idea of understanding idealization as a relation between structures seems to be better motivated if we think of idealization as a relation among theories better than among sentences or statements. Although Nowak's original intuition was not in terms of counterfactuals, it certainly has a counterfactual character that could be formally captured by a modal framework, but no such framework has yet been provided and it is not clear that such a framework, in case it could be provided, can be general enough to be applied to scientific theories. Contrary to this, the structuralist view is a powerful tool of representing scientific theories and it is already available. Finally, the distinction between idealization as model construction and idealization as intertheoretical relation can be shown in a much easier way within the structuralist perspective. The other difficulties have to do with the lack of generality of Nowak's account, which is too restrictive because of the nature of the parameters involved, but this difficulty can also be resolved by modifying the definition in an appropriate way.

Now we focus on the empirical confirmation problem. As we know, the empirical claim of a theory **T** says: $\mathbf{I(T)} \subseteq \mathbf{r}[\mathbf{M(T)}]$. We can ask for a similar condition for idealized theories according to which the idealized theory can be shown to be empirically correct. At least, the following result can be proved: that the empirical claim of an idealized theory follows from a condition imposed on its set of intended applications, namely that there is an expansion of this set which is included in the reduct of the class of the $\mathbf{M^*}$-structures in which the factor p_{i+1} takes the value 0. Let $\mathbf{T}\ id\ \mathbf{T'}$ denote that **T** is an idealization of $\mathbf{T'}$:[32]

Theorem 1. $\mathbf{T}\ id\ \mathbf{T'} \wedge \exists Y(Y \subseteq \mathbf{r}[\mathbf{M^*} - \{m: m \in \mathbf{M_p}(\mathbf{T'}) \wedge p^m_{i+1} = 0\}] \wedge \mathbf{I(T)} \subseteq red[Y]) \rightarrow \mathbf{I(T)} \subseteq [\mathbf{M(T)}]$.

Proof: It is easy if we notice that

(+) $red[\mathbf{M^*} \cap \{m: m \in \mathbf{M_p}(\mathbf{T'}) \wedge p^m_{i+1} = 0\}] \subseteq \mathbf{M(T)}$, because

if $m \in red[\mathbf{M^*} \cap \{m: m \in \mathbf{M_p}(\mathbf{T'}) \wedge p^m_{i+1} = 0\}]$, then there is a $m^* \in \mathbf{M^*}$ such that $p^{m^*}_{i+1} = 0$ and $red(m^*) = m$. Therefore, $m \in \mathbf{M(T)}$.

By (+), $\mathbf{r}[red[\mathbf{M^*} \cap \{m: m \in \mathbf{M_p}(\mathbf{T'}) \wedge p^m_{i+1} = 0\}] \subseteq r[\mathbf{M(T)}]$, whence

(++) $red[\mathbf{r}[\mathbf{M^*} \cap \{m: m \in \mathbf{M_p}(\mathbf{T'}) \wedge p^m_{i+1} = 0\}] \subseteq r[\mathbf{M(T)}]$.

Since by assumption $\mathbf{I(T)} \subseteq red[Y]$ and $red[Y] \subseteq red[[\mathbf{M^*} \cap \{m: m \in \mathbf{M_p}(\mathbf{T'}) \wedge p^m_{i+1} = 0\}]]$,

we obtain by (++): $\mathbf{I(T)} \subseteq [\mathbf{M(T)}]$.

In order to make an account of the approximations, AT_i, of the idealizational statements, we can construct–following the suggestion of Balzer & Zoubek (1994)–a continuos sequence of models in which the value of the parameter introduced by the concretized theory becomes closer and closer to zero (or another constant). Let us only sketch (without giving any precise formalization) how it would look. We should consider the class $\{m_i \in \mathbf{M^*}: \forall \overline{x}\,(p^{m^*}_{i+1})(\overline{x}) = r \wedge |\, r - 0\,| \leq \varepsilon\}$, that is, where $r \in \mathbf{R}$ is a certain real number satisfying the property of being "small enough". For each different $r \in \mathbf{R}$ satisfying this property, we become a sequence of models m_i (of their reducts) converging to some model $m \in \mathbf{M(T)}$. Let **U** be a topology on $Str(\theta)$. Then we can say that there is an "idealizational con-

[32] Compare this result with that of Balzer & Zoubek (1994), Theorem 1.b., p. 68.

vergence" of **T** to **T′** with respect to topology **U** if **T** is an idealization of **T′** in the former sense and there exists a sequence of models $\{m_i\}$ such that each $m_i = \langle D_1,\ldots, R_n, p_{i+1}\rangle \in \mathbf{M}^*$, for each $\bar{x}$ there exists a real number r such that $p_{i+1}(\bar{x}) = r$ and $r \to 0$ with respect to the natural topology in **R**, and $red(m_i) \to m$ with respect to **U**.[33] Furthermore, there is an intimate connection between the empirical claim of preceding models in the sequence and the empirical claim of the most idealized mode. The connection is the following: we can say that the most idealized theory is empirically adequate, even if it is not possible to show this directly, by showing that its concretized versions (the preceding models in the sequence) are empirically adequate. Though I will not show this as a separate theorem, I think it would be easy to show this taking into account the preceding considerations (this is work for another article).

What should this convergence show? Or what should this iterative procedure of successively applying idealizations and concretizations mean? Well, science seems to grow by trying to make the laws and theories we have more precise. Sometimes it is necessary to correct them and empirical investigation leads us to extend the domain of our theories. New theories can be applied to a greater domain of entities and old theories can be shown to be valid for a restricted subclass of this domain when we make certain counterfactual (idealizing) assumptions. Under these ideal conditions, it is possible to somehow derive old theories from the new ones. A paradigmatic case of this seems to be that of Kepler-Newton, which is typically taken as an example of an approximative intertheoretical relation (though also involving idealizing assumptions). The concretization process usually means to formulate tentative hypotheses stating whether a certain (new) factor has or doesn't have an appreciable influence on our computations, on our laws. These concretizations can be made years or centuries after a theory has been formulated, as in the case of Newton trying to derive Kepler's laws from his own gravitational theory. But they can also be made immediately as a way of testing our hypotheses. The idealization-concretization process, understood as the process which leads from idealizations by constructing models to the concretizations which make our theories more accurate, seems to be the essence of scientific method. The accuracy of our theoretical models must be understood as the degree to which our theoretical models approach the data. Idealizations can then be justified by showing how they lead to more accurate theories by the converse relation, i.e., that of concretization. As Laymon puts it:

> [i]f it can be shown that more realistic initial conditions will lead via theory to correspondingly more accurate predictions, then the original highly idealized initial conditions are justified in the sense that they provided the starting point for a successful confirmational process (Laymon 1982, p. 115).

The well-known puzzle of how to confirm idealizations can be solved in its turn

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33 Recall that a sequence in a space Y is a map ϕ: $\mathbf{Z} \to Y$ and that we say that ϕ "converges to" y_0 if $\forall \mathbf{U}(y_0)\ \exists N \forall n$ $n \geq N$: $\phi(n) \in \mathbf{U}$, where $\mathbf{U}(y_0)$ is a neighborhood of y_0. For sequences of reals, $\phi \to y_0$ becomes: $\forall \varepsilon > 0\ \exists N\ \forall n \geq N$: $\phi(n) \in B(y_0, \varepsilon)$, where $B(y_0, \varepsilon)$ is defined as the set $\{y: d(y,y_0) < \varepsilon\}$. See Dugundji (1966), p. 209.

by invoking the concept of approximation: it is argued that it is "impossible" to test idealizations, because their antecedents (ideal conditions) are never realisable. To this objection we can reply: maybe idealizations themselves cannot be directly confirmed, but the statements in which it is stated that the idealized factors *do not assume the limit value*, but *approach to it*, these statements can usually be easily confirmed. We merely need to experimentally reproduce a situation which approaches that of the ideal or limit conditions and try to show whether the idealized law approximately holds. There is another kind of justification for the use of idealizations: they should allow for practical computability. In intertheorical cases, such as the Kepler-Newton and the Newton-Einstein relations, the practical computability allowed by the idealized theory enable us to use the old laws *in some restricted domains* and *to a certain degree of accuracy*. Science always impels us to find the best explanation of a phenomenon. In the present context, this turns into trying to find the best concretized form of a certain law or theory, which is proved to be only approximately true. In many paradigmatic cases of "revolution" in science, we have a new theory which ideally tries (1) to explain the anomalies of the old theory, (2) to explain the same successful empirical consequences as the old theory, (3) to have more predictive power, and (4) if it is possible, to be applicable to a much broader empirical domain of entities. Theories which were well accepted in the past and that are proved to be approximately true according to the new and more precise ones should not be rejected. They can still be applicable in a restricted domain. In these cases, it is desirable that the new theories can provide a theoretical explanation of the fact that the old theory is still successful to some extent. For example, Newton accepted Kepler's laws as empirical generalizations holding only approximately and arrived at his theory of gravitation in part trying to give a theoretical explanation for these laws. And Einstein tried to give a formulation for the gravitation field equations which could explain the successful part of Newton's theory. These considerations seem to me in accordance with Kuipers' (2007) reflections on the relation of idealization to truth approximation (he also understands this notion as a relation between idealized structures).

5. Conclusion

The comparison between the two accounts, Nowak's and that of the structuralist view, shows that Nowak's original formulation in terms of the syntactic view is inadequate for methodological purposes. At the same time, our structuralist reconstruction indicates the relevance of the concept of idealization as a specific relation between structures distinct both from the notion of idealization as model construction and from the concept of approximation. As we have shown, this particular kind of idealization, which Nowak tried to capture with his own analysis, can also be formalized in structuralist terms. In this sense, our work should be added to that of Balzer & Zoubek's (1994), who similarly endorse a structuralist version of Nowak's concept of idealization. In further publications I will provide some historical examples that can be reconstructed according to my own account.

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