Newton da Costa on Non-Reflexive Logics and Identity*

Newton da Costa sobre la lógica no-reflexiva y la identidad

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Abstract

Newton da Costa pioneered first-order systems of so-called non-reflexive logics (NRL). According to those systems of logic, the reflexive law of identity is restricted; it does not apply to every kind of object the system deals with. NRL were later developed into higher-order systems and quasi-set theory by Décio Krause. The main motivation to develop such systems came from non-relativistic quantum mechanics. Intuitively, it is argued that quantum entities somehow "lost their identities", they are *non-individuals*. Non-reflexive logics are the systems employed to formally underpin such a metaphysical interpretation of the theory. In this paper we re-access da Costa's contributions in the light of recent developments on both the metaphysics of quantum mechanics and on the latest developments of the formalism. The result is that even though da Costa had not envisaged any specific metaphysical goals with the first advancement of his system, his discussion may provide for clear insights into the actual debate.

Keywords: non-reflexive logics - identity - individuality - quantum mechanics

Resumen

Newton da Costa es uno de los precursores de los sistemas lógicos de primer orden llamados lógicas no-reflexivas (LNR). Según estos sistemas lógicos, la ley de la identidad reflexiva está restringida; no se aplica a todos los tipos de objetos con los que trata el sistema. La LNR fue desarrollada más tarde por Décio Krause para lenguajes de orden superior y una teoría de cuasi-conjuntos. La principal motivación para el desarrollo de estos sistemas proviene de la mecánica cuántica no-relativista. Intuitivamente, se argumentó que las entidades cuánticas de alguna manera "perdieron sus identidades", siendo *no-individuos*. Las lógicas no-reflexivas son sistemas empleados para dar un fundamento formal a tal interpretación de la metafísica de la teoría. Neste artigo, nós reconsideramos as contribuições de da Costa tendo em vista desenvolvimentos recentes tanto na metafísica da mecânica quântica quanto na compreensão do formalismo. En este artículo, reconsideramos las contribuciones de da Costa en vista de los recientes avances tanto en la metafísica de la mecánica cuántica como en la comprensión del formalismo. El resultado es que, aunque da Costa no había contemplado ningún objetivo metafísico con el desarrollo de su sistema, su discusión proporciona indicios de gran interés para el debate actual.

Palabras clave: lógica no-reflexiva - identidad - individualidad - mecánica cuántica

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> *Probablement, dans l'avenir, nous utiliserons de nouvelles logiques, de même que le physicien d'aujourd'hui a recours à des géométries différéntes de la géométrie euclidienne.*
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> Newton C. A. da Costa (1997), p. 128

1. Introduction

There seems to be a clear relation between what is traditionally called *The Principle of Identity*, the so-called *Reflexive law of identity*, and systems of formal logic called *Non-reflexive logics*. Provided that we define these concepts in the following terms, we may not have great trouble spelling out these relations:

> **Principle of Identity** (PI): every object is identical to itself.
> **Reflexive law of identity** (RI): for every x, x = x.
> **Non-reflexive logic** (NRL): logics in which the reflexive law of identity (RI) is restricted or does not hold.

Notice that while the first principle (PI) is formulated only informally and in a rather vague ontological language, the second one (RI) is a purely syntactical principle that may be formulated in any first-order or higher-order formal language with identity sign. It is usually said that the second principle formalizes the first (*i.e.* is the formal counterpart to the first). Having set that, a non-reflexive logic simply imposes a restriction on the second principle, which by its informal relation with the first, means that the Principle of Identity fails in such languages as well.

It was perhaps Newton da Costa in his (1997, pp. 123-126; first Brazilian edition published in 1980) that first provided for a rigorous treatment of such a relation between those principles. The main motivation came from non-relativistic quantum mechanics. According to some of the founding fathers of the theory, in particular to E. Schrödinger (the *locus classicus* is Schrödinger 1996, pp. 121-122), as we shall see, identity does not make sense for quantum particles in some cases (see French & Krause 2006, Chap. 3, for a historical discussion). The formalization of this claim is precisely encapsulated by non-reflexive logics, so much so that da Costa baptized his system "Schrödinger logics". In these logics, it is claimed, the Principle of Identity fails and with such failure, one of the traditional "Laws of Thought" proves to be not universal, not necessary, not *a priori*, and not self-evident. That is, in some sense, if da Costa's reading of Schrödinger's claim is correct, and if his attempt to relate it to a NRL is adequate, then it seems that according to Schrödinger, quantum mechanics violates one of the so-called laws of thought.

However, furnishing such non-reflexive systems by itself does not address a corresponding metaphysical question: what is the metaphysical status of such entities without identity? In other words: granted that identity may possibly fail, what is the status of something failing PI? What kind of entities are those things? Restrictions on the syntax of a language do not, by themselves, provide an answer to that question. Of course, that is a metaphysical worry, and should be addressed by an accompanying metaphysical view.

In recent discussions on the metaphysics of quantum mechanics these entities are precisely the non-individuals, where *non-individuals* are understood as entities having no identity conditions. It was through the development of Schrödinger logics that French and Krause (2006) related non-individuality with the lack of identity, with non-reflexive systems being the tailor-made formal system to deal with such entities (see French & Krause 2006, Chaps. 7, 8, and for further discussion of metaphysical matters, French 2015 and Arenhart 2017).

Here, however, it is important to keep in mind that two not necessarily related themes end up constantly sandwiched together: on the one hand, the idea that quantum entities are not individuals was a novelty also introduced by some of the founding fathers of the theory, but on the other hand, this metaphysical idea *needs not be formally framed in terms of a lack of identity*. Let us call the metaphysical articulation of non-individuals in terms of lack of identity the *non-reflexive* approach to non-individuals. As we shall see, da Costa may be seen as furnishing an early articulation of such a view too, even

though he does not endorse it explicitly in his (1997). His articulation, in particular, differs in significant respects from those presented in French and Krause (2006), which are taken here as the standard account on the non-reflexive approach.

This paper is divided as follows. In section 2 we review da Costa's system and its main motivations. In particular, we discuss da Costa's understanding of the Principle of Identity, an important issue when it comes to deal with the motivations to have PI violated in his system. In section 3 we discuss a possible understanding of the impact of violating the principle of identity in the light of da Costa's scientific approach to philosophy. In particular, it will become clear that by presenting a non-reflexive logic da Costa places himself inside a naturalistic tradition and explicitly adopts what is nowadays called a *thin* notion of objecthood. Curiously, this approach to non-individuals has rarely been discussed in the context of the metaphysics of quantum mechanics. It differs significantly from more standard (and more metaphysically loaded) approaches such as French and Krause (2006), for instance. Finally, in section 4 we relate the developments of previous section to contemporary discussions on the metaphysics of quantum mechanics. An outcome of the discussion is that da Costa may provide a viable alternative to understand the lack of identity in a system of logic while avoiding some of the problems addressed to this position in current debates. We conclude in section 5.

2. The principle of identity modulo da Costa

Given that da Costa is willing to restrict a version of the traditional (ontological) Principle of Identity in his formulation of Schrödinger logics, it will be useful to discuss how he understands such a principle. As we shall see, there is a major difference here from most traditional discussions about these important issues, a difference having to do with the distinction between *diachronic* and *synchronic* identity. It will be seen that da Costa demands that identity takes care not only of synchronic situations, but also of diachronic ones as well. The exact impact of such a demand we shall briefly discuss by the end of the section.

Let us begin with the conflation of the two notions of identity, diachronic and synchronic. This identification of the two notions will become evident as we examine the quote by Schrödinger with which da Costa motivates his system. Schrödinger's claim, taken in context, encapsulates a clear diachronic element not present in the cited fragment. Schrödinger, as quoted by da Costa in (1997 p. 121), says:

> I beg to emphasize this and I beg you to believe it: it is not a question of our being able to ascertain the identity in some instances and not being able to do so in others. It is beyond doubt that the question of 'sameness', of identity, really and truly has no meaning. (Schrödinger 1996, pp. 121-122)

In the wider context of this quote, Schrödinger is explaining to a general audience the impact of quantum mechanics on our understanding of the nature of atomic particles. According to him, the traditional understanding of atoms as persisting objects with a well-determined trajectory that could be—in principle, at least—constantly followed by us must be completely abandoned in quantum mechanics. His explanation of this fact, however, appeals essentially to a *diachronic* notion of identity. It goes roughly as follows: when one observes a particle at a given place and time (*e.g.* here and now), that observation should be considered as an isolated event. Another observation, even if conduced at a very close place and closely related time, should not be taken to be an observation of *the same particle*, even if there are good reasons to suppose that there is a causal relation between the two events, *i.e.* that it is the same particle that followed a continuous trajectory which could relate both events. So, one loses the main feature of classical atoms: their trajectories being traceable continuously in space and time. According to Schrödinger, by losing a specific trajectory, particles have lost their identity, their individuality. So, *nota bene*: when taken in its full context, it is the claim that particles are the same *in distinct instants of time* that Schrödinger claims to be meaningless; statements of identity in these cases are merely a convenient way of speaking.

So, according to Schrödinger's explanation in this context, quantum mechanics imposes on us that we have only isolated observations of particles here and there, but a continuous trajectory cannot be followed or inferred from such events. As a result, there is nothing in the theory that could grant that we really have an underlying trajectory of the same object. In this sense, linking isolated observations to constitute a continuous trajectory is illicit: it may be only a convenient way of speaking, but truly there is no sense in saying that it is the same particle in each isolated observation that we are seeing. In this sense, Schrödinger is clearly appealing to spatio-temporal continuity as a principle of individuality. That, by itself, is a traditional answer to the problem of individuation, and as a result of quantum theory, it seems, quantum particles fail the principle. In other places Schrödinger does discuss the impact of quantum mechanics on synchronic identity, but da Costa does not appeal to such sources (see Schrödinger 1998, for instance, for a discussion of quantum statistics on identity).

Now, it seems that taking such a diachronic version of identity in the context of a discussion of PI is no mere accident in this case. In fact, da Costa explicitly relates the diachronic discussion with the traditional principle of identity. Recall that we have seen in the previous section that the formulation of the principle—even though da Costa takes it to be inadequate and rather vague—states that "every object is identical to itself". That is, this formulation involves no explicit mention of time; it seems to comprise a synchronic account of identity, and that is precisely how it is traditionally understood. Presupposed in the formulation, one usually finds the proviso that everything is identical to itself at a *given fixed time*. The formulation of PI according to the reflexive law of identity in first-order logic also does not take time explicitly into account. So, how are we to understand the relation between failing a diachronic principle of identity and failing a clearly synchronic principle of identity?

That is a crucial point if we are to understand the nature of the proposal of Schrödinger logics by da Costa. It would clearly be a simple misconstruction if things stood as they are: the reflexive principle of identity captures a synchronic version of identity, while Schrödinger is really claiming that it is (at least) a diachronic version of it that fails. However, da Costa is ready to address that worry. His comments on the deficiencies of PI show that he is *not* willing to restrict the scope of PI to a purely synchronic version. Let us check.

The best evidence as to the fact that he has in mind an unrestricted interpretation of the PI comes from his critique of the traditional statement of the principle. After considering inadequate the traditional formulation of the PI stating that "A is A", da Costa considers the possibility that it states "A = A", where "A" is a variable ranging over objects. In page 101, da Costa (1997) poses the question: "Physical objects change continually, so how can they keep identical to themselves?" (all translations of passages from da Costa 1997 are ours; from now on we shall omit that reminder). That is, the PI is clearly being understood as having to account for identity over time as well. In this sense, its validity is clearly questionable for mid-seize physical objects already.

This same problem then returns some pages later (pp. 120-121). It is precisely here that Schrödinger's claims appear as evidence that the PI's validity may be seriously questioned. However, not only Schrödinger, but even Heraclitus and Lenin are quoted here to contribute with their famous passages to add suspicions to the validity of PI. To illustrate the point, da Costa goes on and asks us to take a person such as Napoleon into account. During his military career Napoleon has clearly not been identical to himself. "He had not always been bald; he was sometimes riding a horse, sometimes not; in his youth he was thin, fattening latter" (1997, p. 121).

It is clear from these cases that da Costa takes the Principle of Identity as inadequate because it does not account for identity *over time*. So, as we have already mentioned from the discussion of the quote by Schrödinger, identity is not restricted to a single instant of time. In fact, da Costa suggests two possible strategies to make the principle apply to reality (1997, p. 121). The first one consists in making identity relative over time, so that a modified version of PI really reads "$A =_t A$, for all A and t". The second strategy consists is considering Napoleon as the spatio-temporal continuum of events constituting his life.

The first strategy is a modification of da Costa's unrestricted notion of identity; the second is an attempt to account for the very nature of objects in order to keep identity fixed. Both fail according to

da Costa. In both cases identity becomes extremely complicated and very unlikely to codify one of the so-called "laws of thought". More revealing, however, is the complaint (1997, p. 122) that any attempt to save PI through the postulation of some kind of substance would involve metaphysical underpinning of a law of logic. As we shall see, that is a path that da Costa will reject due to methodological reasons.

So, the conclusion is that saving a synchronic version of the law of identity requires either metaphysical *ad hoc* moves (positing substances, for instance), a move rejected (among others) on grounds that they are methodologically objectionable, or else shifting to modified versions of the law that are rejected on grounds of their lack of simplicity and the dubious results that seem to follow from them. In this sense, identity must be interpreted as unrestricted, and the PI clearly fails even for mid-sized objects as Napoleon.

There are two clear alternatives here. One is to reject the methodological stance adopted by da Costa and allow that some metaphysics enter the characterization of identity, so that we make a legitimate distinction between synchronic and diachronic identity. In this case, Schrödinger's claim (as well as Heraclitus') about temporal change do not contribute to the failure of the principle of identity, but concern rather identity over time, and da Costa does not seem to have a case against it. In fact, that route would seem to preclude da Costa's motivations for the development of non-reflexive logics as they stand, and we would be able to distinguish the discussions of synchronic and diachronic identity, as they are typically distinguished. Without a case against synchronic identity, then, there would be no case against the PI. Another alternative would be to hold the strict methodological stance da Costa adopts and recognize that identity really fails to apply to objects overall. Curiously, this option, as we shall see briefly, is not adopted even by da Costa. He seems to fail to notice that once the stage is set as he did, there is no hope for saving identity as holding even for the most simple of the objects, not only for quantum objects.

Given that our aim is to present and enlarge da Costa's contribution to the subject, we shall follow the second strategy here, for working purposes. Obviously, as we shall discuss later, the methodological stance adopted by da Costa may raise issues by itself. Also, as we shall mention, da Costa does not follow the conclusion of his position and seems to imply that only quantum objects fail identity.

With that in hand, da Costa's system is free to codify the failure of PI exactly through the failure of the RI (the reflexive law of identity). His Schrödinger logics are two-sorted first-order systems. Besides typical logical apparatus such as connectives and quantifiers, and non-logical terms such as predicate constants, there are two kinds of individual terms (we leave individual constants out for simplicity). The first kind of terms x, y, z,... intentionally denotes quantum objects, and formulas such as t = k are not formulas of the language, for t or k terms of the first species. The second kind of terms X, Y, Z,... denotes classical objects, and for them identity makes sense, so that T = K is a formula, for K and T terms of the second species. Apart from that, the usual logical apparatus of connectives and quantifiers is governed by typical classical postulates (see 1997, pp. 123-126).

As we have mentioned a few paragraphs before, notice the following feature of da Costa's formulation of Schrödinger logics: it is curious that the failure of identity is now ignored for classical objects. Given that even an object such as Napoleon poses difficulties to identity and that any strategy to fix the basic properties of identity is rejected, why keep identity for anything after all? That is a point that deserves further discussion, which we shall not carry on here. It seems that da Costa's argument either go too far to pose problems for identity, leaving identity at best for mathematical (outside space-time, immutable) entities, or else identity should be restricted (with a salutary dose of metaphysics being admitted) and be understood as comprising only the case of synchronic identity. In the second case, as we have already mentioned, non-reflexive logics would not be motivated. Interesting as they are, those questions shall not concern us at this moment.

3. Objects without identity

With Schrödinger logics in mind and the above peculiar understanding of the Principle of Identity whose validity gets restricted in such system, it could be questioned now what kind of metaphysics do we get from such a distinctive view. By focusing only on the formal system itself we get no clue as to how we should understand the nature of the entities that are being dealt with. Does logic provide for such a view? Does da Costa provide for a metaphysics clearly in tune with such a logical system? As we shall discuss in this section, there is no clear articulation of a metaphysics of entities without identity in (1997), but da Costa provides for a general view about the role of logic on the accompanying ontology of a theory wholly compatible with a traditional understanding of objects that goes back at least to Frege and Quine. As it happens, that view on objects finds much approval today even in the philosophy of quantum mechanics; the main novelty here is the extension of the view to accommodate a pluralism on logic and da Costa's view on the development of science and scientific reasoning.

So, to prepare the framework where such a discussion is carried on, let's go for some of the basic conceptual apparatus employed by da Costa in his (1997). It concerns mainly the relation between the use of reason in scientific contexts, and the relations of reason to logic and language. We shall now present roughly how da Costa articulates those concepts. So, even though da Costa does not articulate how Schrödinger logics could possibly relate with entities without identity, his overall explanation on the relation of logic and reality may be useful for us to provide such interpretation.

Reason is characterized in the Introduction of da Costa (1997) as a faculty divided in two main parts: *constitutive reason* and *operative reason*. *Constitutive reason* is the passive aspect of reason and basically furnishes us the *categories*, the key-concepts with which we shape our cognitive experience. For example, general concepts such as object, relation, and property, are certainly among the categories for da Costa. More controversially, however, he also introduces concepts such as causality, space and time, among reason's contribution to shape experience. A case could be made here against making reason responsible for those concepts, but given that they are not central to our discussion we shall not raise such issues here. *Operative reason*, on its turn, is the active part of reason which combines the basic categories allowing us to draw inferences based on such concepts. It is from the workings of operative reason that we get not only basic logical inferences, but also a mathematics; it is also thanks to those inferences we may extend our knowledge further than our unaided immediate experience would allow us.

Focusing now on the constitutive reason, the categories that are provided for receive a *sui generis* treatment by da Costa. They are a blend of the Aristotelian kind of ontological categories and Kantian *a priori* categories of the understanding. Let us briefly explain, because that is where language and experience have a major role to play. In principle, in common day usage such categories are framed in natural language and have a rather vague content. General notions such as object, property, space and time do not have a well-defined meaning in natural language. They have at best a core intuitive meaning. What da Costa does is to propose that we should study the precise meaning those general concepts acquire in specific scientific contexts, which he equates with rational contexts. That is, it is inside the context of a scientific theory that the notion of object, properties, and relations (among other categories) get precise meaning. Perhaps that suggestion gets clearer when we deal with the concepts of space and time: in Newtonian mechanics such concepts have a specific meaning, while in the special theory of relativity they clearly have a distinct meaning. The concepts of space and time are specified inside a scientific theory. Of course, as we mentioned, space and time are very controversial items to be put among the categories of *reason*, but we shall confine ourselves to the notion of object, and this notion shall also derive specific meaning from specific contexts of scientific exposition. It is not that those concepts do not have a meaning outside a scientific theory; their meaning just is not specific enough.

So, in the sense that those concepts get their precise meanings according to the specific (scientific) context in which they are employed, they may vary from context to context. As da Costa (1997, p. 54) sees that point, the very possibility of variation also means that the categories cannot be purely

ontological: ontological categories could not vary from time to time, and could not evolve along with the evolution of scientific theories. Reason, in this sense, is subject to a given period of history. So, some variance in the meaning of the categories must be allowed; they are not definitive and necessary. In the same sense, however, they must somehow reflect the nature of the entities in the context at a given period of scientific research. It is as if the general categories present in ordinary language could go through a precisification when a specific scientific context is selected.

Notice that within that description it is not up to philosophers to frame a set of categories applicable once and for all to every context. Philosophy has not, as a privileged role, the mission to furnish the foundations over which science will be developed. According to da Costa's understanding of scientific philosophy, a scientific philosopher has actual scientific theories as her only source, so no purely *a priori* speculation about the workings of the categories will be allowed. The precisification of the categories is the result of the study of a scientific context where the categories are employed. The categories and the logic governing them are obtained only *a posteriori* in a given context (the main source here is da Costa (1997, Chap. 2, Secs. V and VI). So, again, no hope for a presentation of those categories, in a fully specified way, that could be prior to any scientific theory; as a result, science does not require philosophical foundation, but rather, philosophical investigation requires science to get started, to have a subject matter.

The methodology of the scientific philosopher also strongly suggests that formal languages be employed to the study of specific theories and the creation of thought experiments (thought experiments are here understood in the sense that once a formal system with such and such features is shown to possible, the features encompassed by the system are shown, as a consequence, to be possible). In particular, the axiomatic method with formalized languages is to be employed. According to da Costa,

> Logical principles reflect, in certain aspects, the laws that guide the workings of reason. There is practically no logic-rational activity without a linguistic bond. [...]
> One may say that the laws of reason are susceptible of being obtained, in large measure, by the critical analysis of the contexts of scientific exposition. (da Costa 1997, p. 39)

Here, contexts of scientific exposition are thought of as formalizations of scientific theories, carried much in the spirit of Hilbert's sixth problem. Formal systems allow a rigorous reconstruction of theories, providing for a precise meaning for logical constants (the underlying logic must be made fully explicit in any formalization). Furthermore, the formal languages employed in such constructions allow us to get a clear meaning for our general categories due to a more or less direct correspondence between *syntactical categories* and the *categories of reason*. In fact, the syntactical categories of individual terms (individual variables and individual constants), predicate constants and variables (da Costa accepts higher-order languages in general), and atomic sentence, correspond to the rational categories of object, relation and property, and fact, respectively (see da Costa 1997, p. 53). So, it is the study of the language of a scientific theory, properly formalized with its underlying logic properly spelled out that provides for the precise meaning of categories which, otherwise, suffer from the lack of specification in natural languages.

The relation between logic, language, and reason is thus roughly presented. What is relevant for us is how the categories of reason are shaped somehow by the scientific contexts in which they are embedded. *The very notion of object is subjected to the demands of the context*. Here, following Gonseth and Bachelard, da Costa (1997, p. 120) allows that the notion of object should be endowed with such a plasticity so that there are as many notions of object as there are scientific contexts. Each context has its own logic and language, which on their turn help making specific the contexts' own notion of object.

By doing that, it is the very notion of "object" that gets its meaning changed whenever the logic changes (da Costa is explicit about the impact of such a logical pluralism on the notion of object in p. 120, although he does not explicitly develops the consequences of such a view on what concerns the category of objects without identity). Now, to finally address the questions with which we began this

section, the resulting conception of object is not a metaphysically loaded notion of object, it is said to be a rather logical or formal notion of object. In fact, the concept of object developed by da Costa fits very well what Lowe (1997) calls *the linguistic conception of object*. Very roughly speaking, according to the linguistic concept, an object is characterized precisely in terms of variables and predication. In this sense, it follows the Frege-Quine notion of object.

Obviously, to say that this is a Quinean notion of object would not be far from the truth, but it would not do justice to the relativization to a given language and logic introduced by da Costa, that is, the pluralism allowed by da Costa to play a major role when it comes to study scientific theories. Quine himself seems to be committed to the claim that everything that exists is an object, mainly due to his famous criterion of ontological commitment: what a theory says that there is are the values of variables bounded by existential quantification in sentences which must be true in case the theory is assumed as true (again, see Lowe 1997 for further discussion). What da Costa does (see also his 2002 for a specific treatment of those issues) is to relativize the criterion to non-classical logics and languages, allowing that the criterion applies to every system of logic, classical or not. The impact on the notion of object is clear: change the logic and the notion of object changes as well. What is relevant for our discussion is that one has the main features for characterizing objects left intact, which are stated in terms of predication, variables and quantification; the meaning of the whole category changes, however, as the logic itself changes. The choice of logic, and consequently, the notion of object to be employed, must be a result of analysis of the scientific theory being considered.

When it comes to deal with entities without identity the result is that they are objects, not in the traditional logic, but rather in Schrödinger logics. If we are to accept the suggestion by Schrödinger that quantum entities are such that identity does not make sense for them, then a logic that complies with those features in that context must be provided for. Non-reflexive logics are such logics, and Schrödinger logics are such logics in particular. So, claims that there are no entities without identity would be clearly ruled out as speculative in this philosophical framework: science suggests that objects behave in such and such a way, and scientific philosophers must study the specific scientific context in order to understand such general ideas, in particular through the construction of appropriate formal languages.

Before we finish this discussion, a last remark is in order. Although he recognized the possibility of creating logics where PI is invalid to cope with quantum mechanics, da Costa considered such systems as part of the method of constructing formal systems as thought experiments. If someone claims that it is possible to violate a logical principle, then systems violating such principles must be presented. No claim that quantum mechanics *requires* such systems was made by da Costa, although he clearly sympathizes with efforts to provide for a non-classical logic for quantum mechanics (see 1997, p. 123).

4. Thin objects in quantum mechanics

4.1. Traditional thin objects return

The idea that quantum objects may be seen as thin objects is not new in current discussions of quantum metaphysics. Indeed, thin objects are precisely what is behind the weak discernibility approaches to quantum (in)discernibility, and are also behind a division inside current structural realism in philosophy of science: there is no agreement on whether structural realists should be eliminativists about objects or whether they may accept a thin notion of object (see Muller 2011, French 2014 for further discussion). Here we shall be concerned only with how da Costa's approach may help us illuminate further the debate on quantum entities focusing on weak discernibility. In fact, it may bring a further option to the debate and shed light on some assumptions that underlie much of the discussion concerning weak discernibility.

The recent weak discernibility trend in quantum mechanics and thin objects results from the discovery that the traditional debate concerning quantum discernibility was biased towards ignoring the full power relations provide to distinguish quantum entities (see Bigaj 2015 for general discussions

on weak discernibility; Arenhart 2013 for discussion on what is really achieved). Indeed, as the debate on quantum metaphysics is framed for instance in French and Krause (2006), it seems that we have only two major options: either to accept that quantum entities are completely indiscernible and have no identity conditions, being non-individuals (following Schrödinger's lead), or else to claim that even though quantum particles are indiscernible, they may still be individuals: it is sufficient that their individuality be grounded not on their properties and relations (that is, individuality is not framed in terms of any kind of discernibility), but rather on some kind of substratum or primitive thisness (see French & Krause 2006, Chap. 1, for that distinction).

Faced with non-individuals on one side and metaphysically loaded individuals on the other, we seem to have to acknowledge the fact that those entities are not objects in the traditional, thin, sense of the word. Muller (2011, p. 225) seems to have this suggestion in mind when he states that the typical notion of object, according to the Quinean tradition, framed in classical first-order logic, seems to fail when we deal with quantum entities:

> Let's face it, *indiscernible* objects are no objects at all. Identity conditions break down. No identity, no entity. Quantification breaks down. We can talk no more—and *if* someone wants to keep talking of physical objects nonetheless, then he must kiss standard mathematics and classical logic goodbye, must embrace an entirely different set theory and logic [...].

Of course, Muller here is suggesting that a non-classical logic may provide for an appropriate framework in which to talk about indiscernible objects (and objects without identity). That is precisely what da Costa has achieved with Schrödinger logics, as we have suggested earlier! But before exploring how that approach may contribute to the discussion, it will be fruitful to discuss what Muller really had in mind. It turns out that the suggestion quoted has a decidedly rhetorical purpose: traditionally, it seems quantum entities are indiscernible, have no identity, and the very notion of object is in danger; in fact, we are even required to change logic. However, (and here comes Muller's real point) once weak discernibility is allowed to enter the stage, no change in underlying logic is required, and the Quinean notion of object is indeed vindicated; and more important, as a result, the tradition of indiscernible non-individuals must be abandoned, while the idea of indiscernible individuals individuated by substrata gets unmotivated.

The result is pernicious to non-reflexive logics. If Muller is right, then there is no need to change logic, because quantum entities are in fact objects in the Quinean sense, having even discernibility conditions. We shall see that there is a way out, however. It concerns precisely the role of logic in shaping our notion of object.

Before doing that, let us pause to see how precisely weak discernibility works. Weak discernibility adds a degree of discernibility to the usual notions of discernibility. Typically, we say that objects x and y are *absolutely discernible* when there is a property one of them has that the other does not have. Objects x and y are *relatively discernible* when there is a relation R holding for them in one order but not the other (*e.g.* Rxy but not Ryx, or vice versa). Quantum entities are not discernible in those two degrees. Traditionally, those were the only kinds of discernibility taken into account in the discussions on quantum mechanics. Entities x and y are *weakly discernible* when there is a relation that is symmetric (that is, if Rxy then Ryx) and irreflexive (that is, for no x does Rxx hold) holding between x and y. Now, weak discernibility is said to hold for quantum entities. Fermions in the singlet state, for instance, instantiate "having opposite spin to", a symmetric and irreflexive relation. So, even though properties and relations do not in general distinguish quantum entities, weakly discerning relations do. That means that identity may be preserved for those entities, given that when x and y are weakly discernible it follows from the logic of identity that they are distinct. Also, accompanying that logical innovation there is a metaphysical option not envisaged so far in the debate: quantum entities are *relationals*, entities that are only discernible from each other by weakly discerning relations (see Muller & Saunders 2008, Muller 2011).

That approach, besides saving the traditional thin approach to objects, was also seen by Saunders (2003, 2006), Muller and Saunders (2008) and Muller and Seevinck (2009) as providing for reasons for

us to abandon definitively any idea that quantum entities are non-individuals and that non-reflexive systems of logic are required for them (thus, the claim that Muller's suggestion in the above quote that distinct systems of logic must be employed is rhetorical). In fact, the whole tradition according to which quantum entities have somehow "lost their identities" must be rejected. Quantum entities have identity conditions and are weakly discernible.

Does that put an end to non-reflexive attempts to ground quantum non-individuality? Are Schrödinger logics of mere historical interest, an idea that could work only before we discover weak discernibility? Not really. Here da Costa's approach to logic in general, and to non-reflexive logic in particular, may bring to light a deep methodological disagreement that is not generally acknowledged. Let us check what is at stake.

4.2. Thin objects depend on the logic

In his first attempt at establishing weak discernibility for quantum entities, Saunders (2003) made a methodological use of first-order classical logic and of the Principle of the Identity of Indiscernibles: given that we are not certain of which are the objects of quantum mechanics, it seems reasonable to assume that the entities of the theory are the entities discerned by the physical vocabulary of the theory. That is, given the properties and relations furnished by an interpreted formalism of quantum mechanics, we employ them to check which entities the theory deals with are, and we do that by checking what entities result discernible by that vocabulary. Those are the objects of the theory; they are *tailored to suit the physics*. Entities not discernible are not to be envisaged as indiscernible objects, but rather as not being objects at all.

This methodological use of classical logic and discernibility is clearly in the spirit of the Quinean tradition. Entities having identity conditions are allowed, entities having no identity conditions are not objects (following the "no entity without identity" slogan). However, the presupposition of classical logic as the underlying apparatus has generated some controversy. French and Krause (2006, pp. 169-170) called into question the use of classical first-order logic as the underlying logic to begin with. Obviously, assuming classical logic already commits one with entities with identity, doesn't it? This is the so-called "circularity" objection: if identity is what is really at issue, then one should not assume a system having identity to begin with (see further Bigaj 2015 for more comments).

Muller and Saunders (2008, pp. 542-543) answered that objection by distinguishing two senses that the notion of object may have:

i) **formal objects** are objects as defined in the linguistic view, items that may be quantified over, the values of variables and objects of predication in first-order languages.
ii) **physical objects** are items that may be discerned by means of a non-logical vocabulary comprising only physical magnitudes, the predicates and relations provided by the physical theory under study.

Their claim is that there is no problem assuming that quantum entities are formal objects; that is a metaphysically innocuous assumption. Under that assumption, they then go on and show that quantum entities are physical objects, which is what they claim is really relevant on what concerns identity. However, once we have da Costa's approach we see where the problem with that answer lies. Given that the very notion of a *formal object* already depends on the logic being employed, and given that the classical notion of object is committed with identity (no entity without identity again), one cannot employ such a logic and claim to have established identity for the objects. Identity was already there all the time. It is the classical notion of a formal object itself that is not metaphysically innocuous, contrarily to what is typically claimed (*viz.* that it is a metaphysically thin notion of object, as we have already mentioned).

Notice that this clearly pushes the dispute to the field of metaphysical methodology. Saunders (2003, 2006), and later Muller and Saunders (2008) and Muller and Seevinck (2009), assume classical logic to begin with, and then they go on to establish what objects there are. They seem to believe that

classical logic is neutral as to the very nature of objects and that because those objects are formally characterized, nothing of real importance on their metaphysical features seems to rely on that choice. The result, weak discernibility, however, is rather substantial, and based on that result they claim that a whole tradition must be condemned as plainly wrong.

On the other hand, da Costa makes it clear that it is the empirical theory that should dictate us the notion of object that is involved, and the underlying logic must be developed accordingly. Obviously, da Costa and the rest of the non-reflexive logicians may be wrong as to the nature of those objects, but logic is in the same business of describing reality as physics. It should not be accepted *a priori*; that is, differently than what Muller and Saunders do, one should not accept a logic as given and then proceed to show that the putative objects of the theory satisfy the conditions imposed by such logic on those objects; rather, one should start from the empirical theory and then go on to accommodate the resulting conception of object in an appropriate system of logic. The brief sketch of how logic shapes the notion of object and how that notion depends on the context should have made it clear that logic is not different from physics on what concerns its nature; their difference concerns only their distinct degrees of generality.

So, there is a clear sense in which the choice of a favorite underlying logic and language to a theory has an impact on the resulting ontology. The disagreement between friends of non-reflexivity and friends of weak discernibility may now be carried over to a methodological ground where it may be more clearly framed. Instead of the accusation of begging the question and of discussing Muller and Saunders' answer according to which we should distinguish formal from physical objects, we could now discuss the role of logic in shaping the very general notion of object. It is this discussion that was not already made in the literature, and that may help us clarify what is at stake in those debates.

Is logic *a priori* and everything else rests upon it, or does it depend somehow on the scientific context being studied, so that the very notion of object that will result is also scientifically informed? What da Costa has provided for is an articulation of the second option, one that makes logic more relative to the context and apt to be affected by the results of the empirical sciences. Non-reflexive logics could be once again seen from those lights. It makes them part of the empirical enterprise of gathering knowledge, and also allows that we put the objection to friends of weak discernibility from a clearer point of view. Strangely enough, those considerations are rarely discussed in these contexts. When seen from this point of view, non-reflexive logics are not discarded with the advent of weak discernibility. Rather, it is perfectly in tune with the data furnished by quantum mechanics.

Notice also that the resulting relativization of the notion of thin object to a given logic may provide for an accompanying thin understanding of object. That is, is usually claimed that this Quinean notion of object is not metaphysically demanding (for instance Saunders 2006). It should be contrasted with typical accounts of individuality of objects provided by bundle theories of individuation or by substrata and primitive thisnesses, which are robust metaphysical doctrines of objects (Lowe 1997 also accounts for objects in a decidedly metaphysical theory). The resulting notion of thin objects in Schrödinger logics, then, should also benefit from not being so much metaphysically demanding, it seems to us. Of course, it puts some metaphysical flesh on the notion of object, but does not require that we go as deep as postulating bare particulars and the like.

This should be contrasted with the typical understanding of non-reflexive logics. According to French and Krause (2006, pp. 5, 13-14, 140), the restriction on identity may be understood metaphysically as a failure of traditional substratum theories or primitive thisness. The idea is simple: having a substratum or a thisness is typically understood in terms of self-identity. Socrates' thisness, for instance, is the property "being Socrates". So, in this sense, lack of self-identity may be understood as lack of thisness (or of substratum, depending on one's metaphysics of individuality, of course; for more discussions on that issue, see Arenhart 2017). Now, that is a rather substantial notion of non-individual, it requires failure of a substantial individuality principle.

What a notion of thin object is said to achieve is to avoid such substantial commitments with metaphysical notions of individuality. What we have claimed is that the corresponding notion of thin object in Schrödinger logics may be understood in those terms as well. This by itself does not mean

that it should be preferred; rather, that means that there is another path to be explored on the very idea of non-individuals. It may be more easily demystified once we adopt logical pluralism (as defended by da Costa) and the general tenet that logic is not *a priori*, but rather depends on an interplay between experience and reason.

As we mentioned, this particular understanding of non-individuals is rarely addressed in the literature. Perhaps the notion of thin object has been always so closely tied to classical logic that the very idea of thin objects without identity never seemed to make much sense (except for Muller's quote). However, the notion has been available for a long time, it is closely tied to an interesting conception of logic and its role in science, and now it is up to us to assess the merits of such a view.

5. Conclusion

As we have seen, by meshing da Costa's view on the nature of logic with his motivations for the development of Schrödinger logics, one may obtain a clear picture of the influence of scientific theories in shaping some of the most fundamental concepts we employ to understand reality. In particular, the notion of object is one such fundamental concept. Roughly speaking, the concept of object gets precisified in distinct contexts, and the most appropriate logic for a context helps us getting the meaning of that concept clear. The interplay between experience, language, and logic is to be taken into account.

That notion of object, the one that figures in non-reflexive logics, may be understood as generalizing Quine's notion of object, the usually called *thin* notion of object. As we have discussed, this notion was under discussion recently in the context of quantum mechanics. Apparently, if quantum entities were indiscernible, then they are not objects. On the other hand, if weak discernibility is correct, they are objects, but then they have identity and non-reflexive logics are dispensable. What da Costa provided for is an understanding of logic which allows us to account for objects even without identity; furthermore, that account is clearly naturalistic, it makes even logic depends on empirical science.

Besides furnishing an alternative to weak discernibility in which the notion of objects is kept while identity goes, da Costa also furnishes an alternative to usual interpretations of non-reflexive logics. French and Krause (2006, p. 140), for instance, interpret the lack of identity as representing the lack of a substratum, or of a primitive thisness (for further discussions, see Arenhart 2017). That obviously leads to a metaphysically loaded notion of (lack of) identity. It seems that by sticking with thin objects in Schrödinger logics we manage to introduce an alternative understanding of objects that may dispense use of such heavy metaphysical machinery. This alternative, however, is rarely discussed in the literature.

Notice also that by opening the gate to thin objects in non-classical logics such as Schrödinger logics, da Costa has advanced some way to bring disputes on quantum mechanics to bear on the issue of the nature of logic and on logical pluralism. In our discussion of weak discernibility we saw that Saunders, Muller, and Seevinck, assume classical logic and then go on to determine what objects there are. As we argued, that logical *a priorism* is hardly justified (or, at least, it is not actually justified by friends of weak discernibility). In the age of logical pluralism, granted that da Costa has got it right on the relation between logic and experience, that is a rather risky move. Logic should be more integrated with experience and science, and on those grounds it seems Schrödinger logic is a live option.

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