Truth and Relevancy*

Verdad y relevancia

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Resumen

Hay varios tipos de verdades. En este artículo me centro en las verdades semánticas, y dentro de estas en las fácticas. Estas verdades se atribuyen a enunciados. Repasaré la teoría de la verdad de Bunge y discutiré algunos problemas que la misma presenta. Sugeriré que una teoría de la verdad de los enunciados fácticos debe ser complementada con una teoría de la relevancia, y propondré los postulados básicos de la misma. Finalmente, discutiré brevemente la naturaleza de las proposiciones y el problema de la verdad en las teorías científicas a la luz de las consideraciones semánticas presentadas.

Palabras clave: semántica - Bunge - ciencia - verdad - relevancia

Abstract

There are several types of truths. In this paper I focus on semantic truths, and within these on factual truths. These truths are attributed to statements. I review the theory of the truth proposed by Bunge and discuss some problems that it presents. I suggest that a theory of truth of factual statements should be complemented by a theory of relevance, and propose the basic tenets of it. Finally, I briefly discuss the nature of propositions and the problem of scientific truth in the light of the presented semantic theory.

Keywords: semantics - Bunge - science - truth - relevancy

1. Introduction

"Truth" is a polysemic word. We can differentiate at least two kinds of truths: ontological and semantic. *Ontological truth* is the adequacy of thought to reality. More specifically is a matching of the processes in the brain of a knowing subject to processes in the world. The latter are series of changes that can occur either in the physical environment or in the body, including the brain itself. Ontological truth is then a fact-to-fact correspondence, and should be studied by science, in particular by the neurosciences. *Semantic truth*, on the contrary, is the adequacy of a conceptual object such as a proposition to reality. A proposition asserting the occurrence of an event *e* is said to be true if *e* happens. Semantic truth is attributed to propositions according to some *theory of truth*. Truth is not a property of the proposition if the proposition is factual: there is no analysis of the proposition alone that might reveal whether it is true or not.

Since we can separate propositions into formal (i.e. those of logic and mathematics) and factual ones (i.e. those that refer to facts), semantic truths can also be divided into formal and factual ones.

The elucidation of the concept and criterion of semantic truth corresponds to philosophical semantics. A *truth criterion* should specify a truth *valuation function* that maps propositions into truth-values. This function is a partial function since not all propositions have truth-value. It should be reminded that we are those who attribute values to propositions; so, if we do not do the ascription, the propositions remain neither true nor false. Examples of propositions that lack of truth-value are non-tested hypotheses, undecidable propositions in some formal systems, and untestable propositions such as propositions about singular events inside black holes (e.g. "Dr. Spock smiled after crossing the event horizon"). Note that the same proposition might have truth value for some individuals while not having a definite value for others (as it is the case with the above proposition about Spock: for the people falling along with Spock into the black hole, if any, the proposition has a well-defined truth value; for those remaining outside the event horizon it is impossible to assign a truth value to the proposition).

In short: truth and falsity are not intrinsic properties to factual propositions, but attributes assigned to then on the basis of some evidence.

There is no reason to maintain that there is only one theory of truth that can succeed. If formal and factual truths are of different nature, then we can expect that different theories might apply to formal and factual propositions. In what follows, I present theories for formal and factual truth. Most of what I have to say is based on Bunge (1974a, b) and Bunge (2012), whose work I review and expand. I refer those readers interested in other theories to the current literature. Particularly useful reviews with updated references are given by Mosteller (2014), Burgess and Burgess (2011), and Kirkham (1995). Those interested in degrees of truth will find some outstanding material in Smith (2008).

2. Formal truth

Let *L* be some formal system and *p* a proposition of *L*. We say that the truth-value $V_L(p)$ of *p* in *L* is 1 iff *p* is a theorem in *L*: $L \vdash p$. An abstract formula $\varphi(x)$ in *L* has truth value 1 in *L* iff there is a model of $\varphi(x)$.

If a formal proposition or formula has truth-value 1, we say that they are *true in L*. If a formal proposition or an abstract formula is not true we say that they are *false in* L and we assign them truth-value 0. Examples:

- The proposition '3+5=8' is true in arithmetic of integer numbers.
- The formula 'AB - BC = 0' is true in the arithmetic of integer numbers, but not in the arithmetic of matrices.

The function $V_L(p): \mathcal{B} \to \{0, 1\}$ assign values of 0 or 1 to the set $\mathcal{B} \subset L$ of decidable propositions of L. Notice that undecidable propositions do not have truth value in L, although they might be true or false in a different formal system L'.

In short, formal truth equals either satisfiability or theoremhood. This is essentially Tarski's theory of truth, which is considered sometimes as a theory of correspondence. Actually, it is a theory for the satisfiability of propositions in formal languages.

3. Factual truth

Factual truth is an attribute of propositions concerning facts. We assign a truth-value to a proposition p on the strength of empirical tests such as a run of observations. The assignment is done through a new proposition in the metalanguage: p has a truth value $V_E(p)$ with respect to evidence E. The truth-values can change if the evidence changes. The evidence E is formed by a set of propositions that express empirical determinations of some property M whose value according to p is μ. Then

$$EM = e + \epsilon,$$

where e is the measured value of M and ϵ is the corresponding error. Then, p is true with evidence E if

$$|\mu - e| < \epsilon$$

If we have two different pieces of evidence E and E' we should assign a truth value with the strength corresponding to the evidence of smaller error.

Total truth is rarely known in science. Hence it is desirable to introduce a truth valuation function admitting truth-values others than 0 and 1. We adopt a valuation function of partial truth $V: P \to [0,1]$ that applies a set of propositions to the unit real interval.

The function V is determined by the following postulates (Bunge 2012):

A₁- If p is a quantitative proposition that has been found to be true with the relative error ϵ, then $V(p) = 1 - \epsilon$.

> Example: p = "Blumina is 9 years old'". The actual age is, say, 10 years old.
> Then $\epsilon = 1/10$ and $V(p) = 9/10$.

A₂- If $p \neq \neg q$ for some q, $V(\neg p) = 0$ iff $V(p) = 1$ and $V(\neg p) = 1$ iff $V(p) < 1$. If $p = \neg q$ for some $q \to V(\neg p) = V(q)$.

A₃- For any two propositions p and q, if $p \leftrightarrow q$, then $V(p) = V(q)$.

A₄- If $p \neq \neg q$, then

> $V(p \wedge q) = [V(p) + V(q)]/2,$
> and if $p = \neg q$, then $V(p \wedge q) = 0$.

This can be generalised to any number of propositions p_i, $i = 1, 2,..., n$:

$$V (\wedge_{i=1}^{n} p_I) = \frac{1}{n} \ \sum_{i=1}^{n} V(p_i) .$$

As I discuss below, this is correct only if all propositions have the same relevancy.

A₅- For any two propositions p and q, such as $p \neq \neg q$:

> $V(p \vee q) = \max \{V(p), V(q)\}.$
> Otherwise, $V(p \vee q) = V(q \wedge \neg q) = 1.$

Notice that in the proposed system meaning precedes test since only when we understand a proposition we can test it. In turn, the result of a test leads to an assignation of truth-value. Hence, truth depends on meaning and not the other way around (Bunge 1974b).

4. Relevancy

The theory of factual truth outlined above was developed by Bunge (2010, 2012). It is not free of problems. Let us come back to the example we used to illustrate the axiom A_1: p = "Blumina is 9 years old". If Blumina is actually 10, this statement about the age of Blumina has truth-value of 0.9, i.e. it is approximately true. Let us now consider the following statement, which is almost false: "Blumina is 1 year old". Its truth-value is 0.1. On the contrary, the statement "Blumina is younger than the age of the solar system" is completely true, with a value V = 1. The statement is also completely irrelevant to solve the issue of the age of Blumina, despite it refers to Blumina and her age. We can now draw upon A_4 to arrive at some awkward results.

If p_0 = "Blumina is 1 year old", p_1 = "Blumina is younger than the solar system plus 1 second", p_2 = "Blumina is younger than the solar system plus 1/2 seconds",..., p_n = "Blumina is younger than the solar system + 1/n seconds", then we have $V(p_0)$ = 0.1, and $V(p_i)$ = 1, i = 1,..., n. Thus:

$$V(\wedge_{i=0}^{n} p_i) = \sum_{i=0}^{n} \frac{V(p_i)}{n+1},$$

and,

$$V(\wedge_{i=0}^{\infty} p_i) = \lim_{n\to\infty} \sum_{i=0}^{n} \frac{V(p_i)}{n+1} = 0 + \lim_{n\to\infty} \frac{n}{n+1} = 1.$$

Therefore, the value of the molecular statement is 1, i.e. it is true despite p_0 was false.

With a relevant false statement and a large number of irrelevant true statements we have constructed a true statement. All statements have the same reference.

This result suggests that we should take into account the *relevancy* of the different statements when we are evaluating their contribution to a specific problem.

To this goal I define a relevancy bi-valued function Rel: P → {0,1}. Given a problem F, and a statement p with the same reference as the problem, the relevancy function assigns a value 1 (relevant) or 0 (irrelevant) to p according to:

1. If p expresses a sharp value μ, then Rel p =1.
2. If Rel p ≠ 1 then Rel p = 0.

Then, we can reformulate the postulate A_4 as:

$$V_F(\wedge_{i=0}^{n} p_i) = \frac{1}{n} \sum_{i=0}^{n} \text{Rel } p_i . V(p_i).$$

So now V_F is 0 in our example.

In principle we can propose a generalised relevancy function:

$$\text{Rel}_F : P \to [0,1].$$

This is a function that assigns to each statement a relevancy between 0 and 1 with respect to a problem F. Its explicit form is not general but depends on the specific problematic and the sense of the various statements.

5. Truth bearers and theories

When discussing "the problem of truth", analytical philosophers use to distinguish two different problems: the nature of truth bearers and the truth conditions. I have elaborated above about the truth

conditions for both formal and factual truth. I shall now make some remarks on the objects to which we attribute truth values.

Ontological truth is attributed to thoughts and other processes in the brain. The brain is a physical system that can undergo changes that correlate with changes in the external world or other parts of the brain.

Semantic truth, conversely, is attributed to statements and propositions. I have used these two words interchangeably, but now we will differentiate them.

A *statement* is an illocutionary act that expresses an assertive sentence. The statement is a physical object, either a written sentence that express some state of affairs or an utterance. Now, different statements can express the same fact. For instance, the following true statements share the same meaning:

'The show is white'.

'La nieve es blanca'.

'The colour of the snow is white'.

All these statements refer to snow and all say the same: that it is white. Adopting a specific semantic theory of meaning (Bunge 1974a, b), we can form a concept, a class, with all physical statements of identical meaning. I call such a class a *proposition*:

$$p = \{x: x \text{ Syn } s\},$$

where *s* is some concrete statement and Syn is the operation that assigns to *s* all its synonymous statements *s'*:

$$s \text{ Syn } s' \leftrightarrow \langle R(s), S(s)\rangle = \langle R(s'), S(s')\rangle,$$

where R and S are the reference and sense of *s* (Bunge 1974a).

A proposition is then an equivalence class of statements. Synonymy is the corresponding equivalence relation.[1]

Notice that 1) *p* is a concept, not a physical object, 2) strictly, *p* can be defined only when sense and reference can be consistently calculated, i.e. when *s* belongs to a formalised interpreted language or theory, and 3) that this definition is not that proposed by Bunge (1974a,b), who considers propositions as equivalence classes of thoughts. I do not follow Bunge because it is far from clear to me what is a class of thoughts or which is the equivalence relation between thoughts.

Now, with our definition of proposition we can attribute truth to any statement, and the truth value will be inherited by the corresponding propositions, since statements with the same meaning have the same truth value.

$$\forall x(x \text{ Syn } s) \rightarrow V(x) = V(s).$$

Beliefs are psychological attitudes of attachment to some propositions or systems of propositions. There is not direct link between the truth-value of propositions and that we might attribute to beliefs: anybody can believe false statements and consider as false actually true propositions. The believing brain should be studied by the neurosciences and not by philosophical semantic. Belief should not have any place in neither science nor philosophy[2].

Another important question is whether theories can be true. Theories are hypothetical-deductive systems that are constructed to represent some aspect of reality (e.g. Bunge 1967). Any theory involves an infinite number of statements, in the form of theorems entailed by the axioms plus some complementary assumptions and conditions. Hence, it is *not* possible to establish the truth value of a

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[1] For an early attempt in this direction, see Russell (1940).
[2] The reader can already foresee that I reject the usual definition of knowledge as true belief.

theory from the truth values of the entailed statements. Simply, there is no way to test all statements of a theory since actual infinities do not exist, or, if they exist, supertasks are impossible (Romero 2014). However, it is perfectly possible to determine whether some theory T is *truer* than other theory T' that refers to the same facts. We say that T is truer than T' if the finite number of statements S of T has an average truth value and a lower mean error than the corresponding set S' of T'. For example, Special Relativity is truer than Newtonian mechanics and General Relativity is truer than Special Relativity plus Newton's gravitation theory.

6. Conclusion

Summing up: only some brain processes and statements can be true, false, or something in between. Propositions are constructs that inherit the truth value of the statements from which they are abstracted.

A truth-value cannot be assigned to a theory or to a worldview. A theory, however, can be truer that another. The same holds for worldviews. Science thrives for finding theories ever truer and more relevant about the world and the problems it poses to us.

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